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# Quasiparticle-phonon interaction in non-magic nuclei

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**Abstract.** A general microscopic approach to describe properties of excited states in non-magic nuclei is formulated. It is based on the consistent use of the Green function method in Fermi systems with Cooper pairing. The main attention is paid to even-even nuclei, but for odd nuclei with pairing some important relations are obtained too. The quasiparticle-phonon interaction which is introduced acts also in the particle-particle channel and gives a quasiparticle-phonon contribution to pairing. When applied to the theory of giant multipole resonances, the approach includes all known sources of resonance width, *i.e.* QRPA configurations (which correspond to Landau damping in magic nuclei), the single-particle continuum (escape width) and more complex configurations (spreading width). The use of the Green function method makes it possible to include consistently the ground-state correlations induced by the more complex configurations. In the approximation of the collective phonon creation amplitude squared, which is considered in detail here, these are the ground-state correlations caused by two-quasiparticle–phonon configurations; effects of these correlations have been found earlier to be noticeable for magic nuclei. Such a unified approach will give a reasonable description of the giant resonances' integral characteristics including their widths and of some more delicate properties like fine structure and decay characteristics. Physical arguments and earlier results of a similar approach for magic nuclei allow to use the known parameters of the Landau-Migdal non-separable interaction for all non-magic nuclei (except the light ones). This means that the theory developed is suitable for realistic predictions of the properties of unknown nuclei including unstable ones. The inclusion of the single-particle continuum allows to consider also nuclei with separation energy near zero.

**PACS.** 21.60.-n Nuclear-structure models and methods – 21.30.-x Nuclear forces – 24.30.Cz Giant resonances

## 1 Introduction

One of the most important directions of the modern microscopic nuclear theory is an improvement of the random phase approximation (RPA) for magic nuclei or the quasiparticle random phase approximation (QRPA) for non-magic nuclei. This means taking into account configurations which are more complex than particle-hole or two-quasiparticle excitations, respectively. It is very natural, in general, that the higher the excitation energy the more complex configurations should give a contribution to the level density and to the description of individual levels. From this point of view, we need an improvement of the (Q)RPA for levels' energies beginning from the energies of two-phonon states because, microscopically, one-phonon excitations are (Q)RPA excitations. But probably the most convincing evidence that it is necessary to go beyond (Q)RPA has been given by giant resonance physics.

It is well known that the RPA is even not able to describe all the integral characteristics of giant resonances, first of all their widths and sometimes mean energies. It is also known that for medium-mass and heavy nuclei the main contribution to the width is mainly given by two-particle–two-hole or one-particle–one-hole–phonon configurations, see, for example, the surveys [1–6]. These and many other questions have been clarified considerably by two approaches for magic nuclei which took into account the three main mechanisms of resonance damping in a finite nucleus: decay via particle-hole configurations of discrete spectrum (Landau damping, or width), decay via particle-hole configurations with a particle in the continuum (escape width) and decay via more complicated configurations of the two-quasiparticle–phonon type (spreading width). One of these two approaches [7–9] is self-consistent on the RPA level, *i.e.* instead of two sets of

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parameters describing the mean field and the interaction of nucleons, only one set of the Skyrme force's parameters was used. The second approach [5,10–16] is based on the consistent use of the Green function method and is a generalization of the standard Theory of Finite Fermi Systems (TFFS) [17] to include the one-particle–one-hole–phonon states and the single-particle continuum. Both approaches used realistic non-separable forces and were realized numerically for 3–8 doubly magic nuclei including the unstable ones [9,11–13]. Inclusion of the single-particle continuum is particularly important for the widths of high-lying giant resonances, for the unstable nuclei with the nucleon separation energy near zero [9] and for more delicate problems, such as the description of the resonance fine structure and decay characteristics [5,8,13,15].

The question arises about many hundreds of non-magic nuclei which are of great interest in astrophysics and other fields. In fact, there are no similar giant resonance calculations with non-separable forces in such nuclei. No doubt, these nuclei also require a consistent quantitative treatment and predictions of their characteristics, *e.g.* for unstable non-magic nuclei which can be studied now experimentally with radioactive beam facilities.

During the last years many experimental results have been obtained using the new generation of gamma-ray detectors, namely the germanium ones, EUROBALL Cluster, EUROGAM etc. [18,19]. High resolution and unprecedented efficiency for the detection of high-energy photons up to 20 MeV has already given much new information not only about superdeformed bands but also about characteristics of one-quasiparticle–phonon, one-quasiparticle–two-phonon multiplets in odd (see, for example, [20]) and two-phonon multiplets in even-even spherical and deformed nuclei. It is clear now that these results require improved microscopic approaches and in the nearest future the number of results will rapidly increase. One can say that these detectors will give direct information about complex configurations which contain phonons. Measurements of similar quality of the excitations above the nucleon separation energy, *i.e.* of the giant resonance fine structure, should be possible too.

Thus, for non-magic nuclei we need a microscopic approach with realistic non-separable forces and complex configurations which should satisfy at least two criteria: i) it should be not too involved numerically, taking into account the huge amount of desirable calculations, and ii) its parameters should be universal (or at least sufficiently well known) so that the predictions of the approach would be reliable enough. The approaches which contain phonons in complex configurations, *i.e.* take into account explicitely the quasiparticle-phonon interaction satisfy the first criterion, because using two-quasiparticle–phonon and two-phonon configurations instead of "pure" four-quasiparticle ones reduces the orders of matrices considerably. This is the *first motivation* of the present work. In addition, such approaches are more transparent physically because, at least for magic and semi-magic nuclei they contain a small parameter $g^2$, where $g$ is the dimensionless creation amplitude of the low-lying phonon. The Hamiltonian approach with two-quasiparticle–phonon and two-phonon configurations in non-magic nuclei was realized in the works of Soloviev and his coworkers [1] who, however, used separable forces to decrease the numerical difficulties of the problem, did not take the single-particle continuum into account and accounted for the ground-state correlations only partially, *i.e.* mainly on the QRPA level. Very recently, calculations of the dipole response in the non-magic nuclei $^{18,20}$O have been performed in the framework of the QRPA plus two-quasiparticle–phonon approach with the Skyrme effective interaction, but not accounting for the continuum [21]. The latter should be important for such light nuclei.

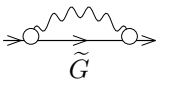
For magic nuclei, the microscopic approach based on consistent use of the Green function method in connection with the $g^2$ approximation in the propagators of the integral equations made it possible to take into account, in addition to inclusion of one-particle–one-hole–phonon configurations, effects of the single-particle continuum [10–16] and all the ground-state correlations, first of all the most interesting ones caused by the complex configurations under consideration [10–16,22]. A reasonable description of characteristics, including widths, of isovector $E1$ [10], $M1$ [12,22], isoscalar $E0$ [14,16] and other giant resonances in magic stable and unstable [12,13] nuclei has been obtained. The method used the simple non-separable phenomenological interaction of Landau-Migdal type with essentially the same and known parameters for all nuclei (except the light ones) and low-lying phonons which have been calculated microscopically. So, in accordance with the basic concepts of the TFFS approach, a generalization of the method to non-magic nuclei (to nuclei which allow the $g^2$ approximation, to be exact) will conserve the same property of the universality of the TFFS parameters. In other words, such a generalization satisfies our second criterion about the reliability of the theoretical predictions and this is the *second motivation* of the present work. Of course, because of non-self-consistency of the approach this criterion should be considered as a first step in the right direction. It should be noted, however, that at present for non-magic nuclei there is not even a formulation of a self-consistent approach with complex configurations like the self-consistent RPA.

During last years there was a discussion about the effects of the ground-state correlations beyond the RPA [13,16,22,23] or QRPA [24], *i.e.* the ground-state correlations caused by complex configurations. It was shown that these effects play a noticeable, sometimes decisive role in the theoretical description of the experimental data. The most striking example obtained within the Green function approach is the explanation [22] of the observed $M1$ excitations in $^{40}$Ca with energy of about 10 MeV and in $^{16}$O with energy about 16 MeV by only these ground-state correlations because the RPA does not give $1^+$ levels in these regions. There are also other dynamic effects of these ground-state correlations in magic nuclei which were mostly experimentally confirmed [5,13,14,16]. The main physical reason of this is that in the Green function approach with one-particle–one-hole–phonon config-

urations a larger class of diagrams has been included as compared with the Extended Second RPA (see [5]), where the 2p2h ground-state correlations have been taken into account and the first order of perturbation theory in the nucleon-nucleon interaction has been used. It is of great interest to investigate these new ground-state correlation effects in non-magic nuclei. This is the *third motivation* to generalize for these nuclei the approach which was developed earlier for magic ones.

In the Green function language, for nuclei without pairing, using the $g^2$ approximation means the explicit treatment of the following one-particle mass operator:

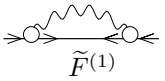
$$M = \quad , \tag{1}$$

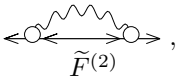



where the circle is the phonon creation amplitude $g$ and the Green function $\tilde{G}$ does not contain the $g^2$ terms. The quantity $M$ results in the excitations which are superpositions of one-particle (one-hole)–phonon configurations in an odd nucleus. In even-even nuclei, correspondingly, there is the coupling to one-particle–one-hole–phonon configurations which, in graphic language, corresponds to "insertion" (self-energy) and "cross phonon" (phonon exchange) graphs [2,25].

The main difference of non-magic nuclei from magic ones is the necessity to take into account Cooper pairing in the nuclear ground state. This means that, in addition to (1), we should also consider the following simplest energy-dependent "anomalous" mass operators [26]:

$$M^{(1)} = \quad , \qquad M^{(2)} = \quad , \tag{2}$$




where the lines with two ingoing or two outgoing arrows denote the anomalous Green functions $\tilde{F}^{(1)}$ and $\tilde{F}^{(2)}$, respectively, which are proportional to the gap. Pairing phonons are not included here because their contribution is expected to be small.

It is implied in the following that the initial quantities of our problem are an "observed" mean field described by a phenomenological potential, *e.g.*, by a Woods-Saxon well, and the pairing gap. The corresponding single-particle levels should be extracted from the observed excitation energies of non-magic nuclei. The initial pairing gaps' values are taken from experiment or from the solution of the BCS gap equation with the phenomenologically determined particle-particle interaction.

For even-even nuclei, by analogy with magic nuclei the graphs (2) lead to the corresponding self-energy insertions and phonon exchange graphs for the excited states (see below). However, the contribution of graphs (2) to the particle-particle channel should be considered also for the ground state, first of all for the pairing problem. As long as the nuclear pairing problem is solved within the BCS approach, the quasiparticle-phonon interaction is not considered explicitely and quantitatively on this level. If we consider explicitely the contribution of the graphs (2) we must also take them into account in the pairing problem and, therefore, to avoid double counting, change the phenomenological particle-particle interaction entering the usual BCS problem. This question will be considered elsewhere but here we use a simpler phenomenological procedure of "refining" the gap values from the quasiparticle-phonon interaction under consideration. It is analogous to the refinement of the phenomenological single-particle energies in magic nuclei which is usual practice in the Green function approach [5,22,25].

This article is devoted to the application of the Green function method to describe characteristics of excited states in non-magic nuclei using a non-separable particle-hole as well as particle-particle interaction in a consistent treatment of QRPA and two-quasiparticle–phonon configurations including the ground-state correlation effects. The main attention is paid to even-even nuclei but for odd nuclei with pairing some useful relations are obtained too (in sect. 3). This article is a continuation of ref. [22] which was devoted to magic nuclei. The generalization to non-magic nuclei is not simple because phonon exchange can significantly contribute to pairing. Therefore the explicit inclusion of complex configurations, phonon exchange in particular, requires re-shaping of the BCS pairing theory too. It is important to deal with the quasiparticle-phonon interaction in the particle-particle channel on the same footing both for the static (pairing, first of all) and dynamical cases.

The article is organized in the following way: In sect. 2 a short derivation of the full system of the QRPA equations is given in the Green function language. In sect. 3 we go beyond the QRPA and introduce, in addition to the particle-hole and particle-particle interactions, a quasiparticle-phonon interaction in a very general form. General equations for one-particle Green functions in non-magic nuclei are obtained which are used in even-even nuclei and also will allow to calculate excitations in odd nuclei with pairing. In sect. 4 the $g^2$ approximation is introduced and the relations which take into account the graphs (1) and (2) are derived for the case that dynamical effects in the particle-particle channel are negligible. Section 5 is devoted to the inclusion of the single-particle continuum. In sect. 6 we discuss the main equations. A simple model calculation is done in sect. 7. Conclusions are drawn in sect. 8 and in the Appendix the formulae for the propagators of our integral equations are given.

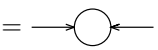
## 2 The full system of the QRPA equations in non-magic nuclei derived in the Green function language

To begin with, we will briefly derive the full system of the QRPA equations for four effective fields in nuclei with Cooper pairing using the Green function formalism, *i.e.* the TFFS approach [17].

The existence of pairing in a Fermi system means that, due to the particle-particle interaction, in the ground state there is a non-zero amplitude for the transition of a parti-

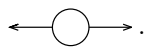
cle in a system of $N$ particles into a hole and a correlated pair in a system of $N+2$ particles

$$\mathrm{i}\Delta^{(1)} = \quad (3)$$




and the same for the reverse process

$$\mathrm{i}\Delta^{(2)} = \quad . \quad (4)$$




We will consider the case

$$\Delta^{(1)*}(N) = \Delta^{(2)}(N+2) \approx \Delta^{(2)}(N) \equiv \Delta\,, \quad (5)$$

which is suitable in practice for all nuclei except for the next-neighbours of magic ones.

Now we must fix the form of the mass operator $\Sigma_{\mathrm{C}}$ which accounts for Cooper pairing. To describe it in the Green function language we take [17]

$$\Sigma_{\mathrm{C}} = -\Delta^{(1)} G^{\mathrm{h}} \Delta^{(2)}, \quad (6)$$

where $G^{\mathrm{h}}$ is the hole Green function without pairing.

For nuclei with pairing it is convenient to use four one-particle Green functions, $G_{\mathrm{C}}$, $G^{\mathrm{h}}_{\mathrm{C}}$, $F^{(1)}$, and $F^{(2)}$. They satisfy Gorkov's equations which in turn are obtained using eq. (6):

$$\begin{aligned} G_{\mathrm{C}} &= G - G\Delta^{(1)} F^{(2)}\,, & F^{(1)} &= G_{\mathrm{C}} \Delta^{(1)} G^{\mathrm{h}}\,, \\ G^{\mathrm{h}}_{\mathrm{C}} &= G^{\mathrm{h}} - G^{\mathrm{h}} \Delta^{(2)} F^{(1)}\,, & F^{(2)} &= G^{\mathrm{h}} \Delta^{(2)} G_{\mathrm{C}}\,. \end{aligned} \quad (7)$$

For finite nuclei, in the representation of the single-particle wave functions $\phi_\lambda$ ($\lambda$ representation) the pole parts of these Green functions are obtained from (7) (except for the case of the odd particle in odd nuclei):

$$\begin{aligned} G_{\mathrm{C}\lambda}(\varepsilon) &= G^{\mathrm{h}}_{\mathrm{C}\lambda}(-\varepsilon) = \frac{u_\lambda^2}{\varepsilon - E_\lambda + \mathrm{i}\delta} + \frac{v_\lambda^2}{\varepsilon + E_\lambda - \mathrm{i}\delta}\,, \\ F^{(1)}_\lambda(\varepsilon) &= -\frac{\Delta_\lambda}{2E_\lambda}\left(\frac{1}{\varepsilon - E_\lambda + \mathrm{i}\delta} - \frac{1}{\varepsilon + E_\lambda - \mathrm{i}\delta}\right), \\ F^{(2)}_\lambda(\varepsilon) &= F^{(1)}_\lambda(\varepsilon)\,, \end{aligned} \quad (8)$$

where

$$\begin{aligned} E_\lambda &= \sqrt{(\varepsilon_\lambda - \mu)^2 + \Delta_\lambda^2}\,, \\ u_\lambda^2 &= 1 - v_\lambda^2 = [E_\lambda + (\varepsilon_\lambda - \mu)]/(2E_\lambda)\,. \end{aligned} \quad (9)$$

The approximation of diagonal pairing $\Delta_{\lambda\lambda'} = \Delta_\lambda \delta_{\bar\lambda\lambda'}$ has been used here. The quantity $\Delta_\lambda$ satisfies the BCS gap equation which contains the effective particle-particle interaction $F^{\xi}_{\lambda\bar\lambda\lambda'\bar\lambda'}$.

To describe the interaction of nuclei with external fields and also to obtain the description of excited states in nuclei with pairing, effective fields (or vertices) are introduced which are changes of the mean field and the pairing gap in the external field $V^0$

$$\begin{aligned} V &= V^0 + \delta\Sigma = V^0 + \mathcal{U}\delta G_{\mathrm{C}}\,, \\ V^{\mathrm{h}} &= V^{0\mathrm{h}} + \delta\Sigma^{\mathrm{h}} = V^{0\mathrm{h}} + \mathcal{U}^{\mathrm{h}}\delta G^{\mathrm{h}}_{\mathrm{C}}\,, \\ d^{(1)} &= \delta\Delta^{(1)} = -\mathcal{V}\delta F^{(1)}\,, \\ d^{(2)} &= \delta\Delta^{(2)} = -\mathcal{V}\delta F^{(2)}\,, \end{aligned} \quad (10)$$

where $\mathcal{U}$ and $\mathcal{V}$ are the blocks which cannot be divided into parts connected by two lines along the particle-hole and particle-particle channels, respectively (we consider here only such external fields which do not change the number of particles, *i.e.* $d^{(1)0} = d^{(2)0} = 0$).

Let us find the quantities $\delta G_{\mathrm{C}}$, $\delta G^{\mathrm{h}}_{\mathrm{C}}$, $\delta F^{(1)}$, and $\delta F^{(2)}$. For example, $\delta F^{(1)}$,

$$\begin{aligned} \delta F^{(1)} = \delta(G_{\mathrm{C}}\Delta^{(1)}G^{\mathrm{h}}) = {} & \\ & G_{\mathrm{C}}(V + \delta\Sigma_{\mathrm{C}})G_{\mathrm{C}}\Delta^{(1)}G^{\mathrm{h}} + G_{\mathrm{C}}d^{(1)}G^{\mathrm{h}} \\ & + G_{\mathrm{C}}\Delta^{(1)}G^{\mathrm{h}}V^{\mathrm{h}}G^{\mathrm{h}}\,. \end{aligned} \quad (11)$$

Here we used the relationship

$$\delta G_{\mathrm{C}} = G_{\mathrm{C}}(V + \delta\Sigma_{\mathrm{C}})G_{\mathrm{C}}\,, \quad (12)$$

which is easily derived from the Dyson equation

$$G_{\mathrm{C}} = G + G\Sigma_{\mathrm{C}}G_{\mathrm{C}}\,. \quad (13)$$

The vertex $V$ determining the change of $G$ in the field $V^0$ was introduced in eq. (12)

$$\delta G = GVG\,. \quad (14)$$

For the hole quantities $G^{\mathrm{h}}_{\mathrm{C}}$, $G^{\mathrm{h}}$, $V^{\mathrm{h}}$ we have similar expressions.

Applying eqs. (7) for $F^{(1)}$ and expressing the Green function $G^{\mathrm{h}}$ by $G^{\mathrm{h}}_{\mathrm{C}}$, we have

$$\begin{aligned} \delta F^{(1)} = {} & G_{\mathrm{C}}VF^{(1)} + F^{(1)}V^{\mathrm{h}}G^{\mathrm{h}}_{\mathrm{C}} \\ & + G_{\mathrm{C}}d^{(1)}G^{\mathrm{h}}_{\mathrm{C}} - F^{(1)}d^{(2)}F^{(1)}\,. \end{aligned} \quad (15)$$

Similary we obtain for $\delta G_{\mathrm{C}}$

$$\begin{aligned} \delta G_{\mathrm{C}} = {} & G_{\mathrm{C}}VG_{\mathrm{C}} - F^{(1)}V^{\mathrm{h}}F^{(2)} \\ & - G_{\mathrm{C}}d^{(1)}F^{(2)} - F^{(1)}d^{(2)}G_{\mathrm{C}}\,. \end{aligned} \quad (16)$$

By analogy with (15) and (16), $\delta F^{(2)}$ and $\delta G^{\mathrm{h}}_{\mathrm{C}}$ have the form

$$\begin{aligned} \delta F^{(2)} = {} & F^{(2)}VG_{\mathrm{C}} + G^{\mathrm{h}}_{\mathrm{C}}V^{\mathrm{h}}F^{(2)} \\ & - F^{(2)}d^{(1)}F^{(2)} + G^{\mathrm{h}}_{\mathrm{C}}d^{(2)}G_{\mathrm{C}} \end{aligned} \quad (17)$$

$$\begin{aligned} \delta G^{\mathrm{h}}_{\mathrm{C}} = {} & -F^{(2)}VF^{(1)} + G^{\mathrm{h}}_{\mathrm{C}}V^{\mathrm{h}}G^{\mathrm{h}}_{\mathrm{C}} \\ & - F^{(2)}d^{(1)}G^{\mathrm{h}}_{\mathrm{C}} - G^{\mathrm{h}}_{\mathrm{C}}d^{(2)}F^{(1)}\,. \end{aligned} \quad (18)$$

The relations (15)-(18) have been obtained in [17] by a somewhat different method.

Substituting the quantities $\delta G_{\rm C}$, $\delta G^{\rm h}_{\rm C}$, $\delta F^{(1)}$, $\delta F^{(2)}$ into eqs. (10) and applying the renormalization procedure described in ref. [17] and using eqs. (8), we obtain the full system of the TFFS equations for four vertices

$$
\begin{aligned}
V &= e_q V^0 + F \int \frac{{\rm d}\varepsilon}{2\pi {\rm i}} \{ G_{\rm C} V G_{\rm C} - F^{(1)} V^{\rm h} F^{(2)} \\
&\quad - G_{\rm C} d^{(1)} F^{(2)} - F^{(1)} d^{(2)} G_{\rm C} \} \,, \\
V^{\rm h} &= e_q V^{0{\rm h}} + F \int \frac{{\rm d}\varepsilon}{2\pi {\rm i}} \{ -F^{(2)} V F^{(1)} + G^{\rm h}_{\rm C} V^{\rm h} G^{\rm h}_{\rm C} \\
&\quad - F^{(2)} d^{(1)} G^{\rm h}_{\rm C} - G^{\rm h}_{\rm C} d^{(2)} F^{(1)} \} \,, \\
d^{(1)} &= -F^{\xi} \int \frac{{\rm d}\varepsilon}{2\pi {\rm i}} \{ G_{\rm C} V F^{(1)} + F^{(1)} V^{\rm h} G^{\rm h}_{\rm C} \\
&\quad + G_{\rm C} d^{(1)} G^{\rm h}_{\rm C} - F^{(1)} d^{(2)} F^{(1)} \} \,, \\
d^{(2)} &= -F^{\xi} \int \frac{{\rm d}\varepsilon}{2\pi {\rm i}} \{ F^{(2)} V G_{\rm C} + G^{\rm h}_{\rm C} V^{\rm h} F^{(2)} \\
&\quad - F^{(2)} d^{(1)} F^{(2)} + G^{\rm h}_{\rm C} d^{(2)} G_{\rm C} \} \,, \qquad (19)
\end{aligned}
$$

where $F$ and $F^{\xi}$ are the effective particle-hole and particle-particle local interactions, respectively, the parameters of which should be obtained from experiment. These parameters are known now [27], see also [10,11]. The quantities $e_q$ are local charges of quasiparticles, they can be obtained from the theory [17] (electric charges) or from experiment (magnetic charges). Parameters of $e_q$ are also known, see for example refs. [10,11].

The excitation energies are obtained from the system of homogeneous equations for the residues of the vertices. The transition probabilities are determined by the residues of the polarization operator at the excitation energies. For details and discussion see refs. [17,28].

## 3 General relations

As was already mentioned, in non-magic nuclei, the conventional causal Green functions $G_{\rm C}$ and $G^{\rm h}_{\rm C}$ must be supplemented by two anomalous Green functions which are caused by the Bose condensate of Cooper pairs. If only the mean field and the particle-particle interaction which gives a pairing gap satisfying the BCS equation are taken into account, all these Green functions are determined by the set of Gorkov's equations. Solutions of these equations are well known for both infinite [29] and finite [17] Fermi systems, as we have seen in sect. 2. As a rule, in addition to this simple particle-particle interaction, it is necessary to take into account the particle-hole and the quasiparticle-phonon interactions. In this case it is very convenient to have equations for the one-particle Green functions in such a form that the well-known Gorkov's Green functions would play the role of free Green functions and simultaneously be the zero-order approximation in the problem of accounting for the additional interaction under consideration. In the Hamiltonian language, this corresponds to the introduction of Bogolyubov's quasiparticles. We will also see that the initial phenomenological mean field, particle-particle interaction and (or) the corresponding BCS gap should be corrected ("refined") in order to avoid double counting of the quasiparticle-phonon interaction. We shall implement in sect. 3 these procedures for a general set of equations for exact one-particle Green functions in Fermi systems with Cooper pairing. The general equations for effective fields in such systems will be also considered in this section. Symbolic notation is used everywhere which is easily spelled out [29,17].

### 3.1 Equations for one-particle Green functions in non-magic nuclei

We consider only two Green functions in an $N$-particle Fermi system (the notation and conventions are as in [29, 17])

$$
\begin{aligned}
G_{\rm C} &= -{\rm i} \langle N | T \psi(x) \psi^{\dagger}(x') | N \rangle \,, \\
F^{(2)} &= \langle N+2 | T \psi^{\dagger}(x) \psi^{\dagger}(x') | N \rangle \,. \qquad (20)
\end{aligned}
$$

They satisfy the set of equations [29]

$$
\begin{aligned}
G_{\rm C} &= G_0 + G_0 \Sigma G_{\rm C} - G_0 \Sigma^{(1)} F^{(2)} \,, \\
F^{(2)} &= G^{\rm h}_0 \Sigma^{\rm h} F^{(2)} + G^{\rm h}_0 \Sigma^{(2)} G_{\rm C} \,, \qquad (21)
\end{aligned}
$$

which generalize the Dyson equation to the case of pairing. Here, $\Sigma$, $\Sigma^{\rm h}$, $\Sigma^{(1)}$, and $\Sigma^{(2)}$ are the corresponding fully irreducible self-energy parts (mass operators), $G_0$ and $G^{\rm h}_0$ refer to an ideal Fermi gas. The equations for $G^{\rm h}_{\rm C}$ and $F^{(1)}$ which describe the inverse processes have similar forms.

In order to obtain realistic equations it is very convenient to single out the well-known ingredients, *i.e.* mean field and Cooper pairing which is described by a BCS-type equation. To do that, we separate each of the full mass operators into two terms, where the first does not depend on energy and the second depends, but is not defined so far

$$
\begin{aligned}
\Sigma(\varepsilon) &= \widetilde{\Sigma} + M(\varepsilon) \,, \qquad \Sigma^{(1)}(\varepsilon) = \widetilde{\Sigma}^{(1)} + M^{(1)}(\varepsilon) \,, \\
\Sigma^{\rm h}(\varepsilon) &= \widetilde{\Sigma}^{\rm h} + M^{\rm h}(\varepsilon) \,, \quad \Sigma^{(2)}(\varepsilon) = \widetilde{\Sigma}^{(2)} + M^{(2)}(\varepsilon) \,. \qquad (22)
\end{aligned}
$$

Here $\widetilde{\Sigma}$, $\widetilde{\Sigma}^{\rm h}$ correspond to a mean field and $\widetilde{\Sigma}^{(1)}$, $\widetilde{\Sigma}^{(2)}$ correspond to the pairing which is described by a mechanism like the BCS one.

In order to compare the approach under consideration with the available experimental data, everywhere in the following we bear in mind the usual situation for a realistic nuclear microscopic theory. Namely, the initial and known quantities are an "observed" mean field, which is described by a phenomenological potential, usually the Woods-Saxon one, and an observed gap, which can be taken from experiment or satisfy the BCS equation with the phenomenological particle-particle interaction.

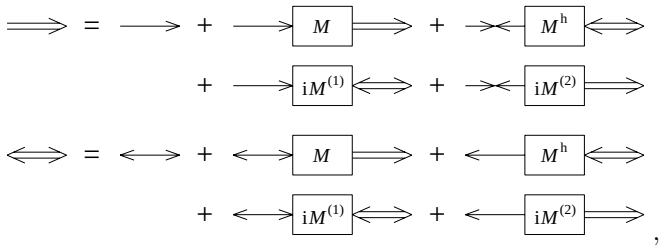
The main goal of our approach is to go beyond the mean field and usual pairing, *i.e.* to take into account the

quantities $M$, $M^{\rm h}$, $M^{(1)}$, and $M^{(2)}$ (hereafter $M^i$) which are supposed to contain the quasiparticle-phonon interaction. They contribute to the above-mentioned phenomenological quantities. Therefore, to avoid double counting it is necessary to exclude their contribution from the phenomenological quantities, or to "refine" the latter from the quasiparticle-phonon interaction under consideration. These refined quantities are marked with a tilde "~". The procedure to obtain the refined single-particle energies and gaps will be described in subsect. 3.4.

It was shown in [26,30] that using eq. (22) one can transform eqs. (21) to the desirable form

$$\begin{aligned} G_{\rm C} &= \widetilde{G}_{\rm C} + \widetilde{G}_{\rm C} M G_{\rm C} - \widetilde{F}^{(1)} M^{\rm h} F^{(2)} \\ &\quad - \widetilde{G}_{\rm C} M^{(1)} F^{(2)} - \widetilde{F}^{(1)} M^{(2)} G_{\rm C}\,, \\ F^{(2)} &= \widetilde{F}^{(2)} + \widetilde{F}^{(2)} M G_{\rm C} + \widetilde{G}^{\rm h}_{\rm C} M^{\rm h} F^{(2)} \\ &\quad - \widetilde{F}^{(2)} M^{(1)} F^{(2)} + \widetilde{G}^{\rm h}_{\rm C} M^{(2)} G_{\rm C}\,, \end{aligned} \tag{23}$$

or, in graphic form




where the rectangular blocks stand for the mass operators $M^i$ which are not specified so far. In the derivation of eqs. (23) the equations determining the Gorkov's Green functions which contain the refined single-particle energies $\tilde{\varepsilon}_\lambda$ and gaps $\widetilde{\Delta}_\lambda$ have been used

$$\begin{aligned} &\widetilde{G}_{\rm C} = \widetilde{G} - \widetilde{G}\widetilde{\Delta}^{(1)}\widetilde{F}^{(2)}\,, \qquad \widetilde{F}^{(1)} = \widetilde{G}\widetilde{\Delta}^{(1)}\widetilde{G}^{\rm h}_{\rm C}\,, \\ &\widetilde{G}^{\rm h}_{\rm C} = \widetilde{G}^{\rm h} - \widetilde{G}^{\rm h}\widetilde{\Delta}^{(2)}\widetilde{F}^{(1)}\,, \qquad \widetilde{F}^{(2)} = \widetilde{G}^{\rm h}\widetilde{\Delta}^{(2)}\widetilde{G}_{\rm C}\,. \end{aligned} \tag{24}$$

The Green functions $\widetilde{G}$ and $\widetilde{G}^{\rm h}$ which determine the new single-particle basis, *i.e.* "refined" single-particle energies $\tilde{\varepsilon}_\lambda$ and single-particle wave functions $\tilde{\phi}_\lambda$, were introduced by

$$\widetilde{G} = G_0 + G_0\widetilde{\Sigma}\widetilde{G}\,, \qquad \widetilde{G}^{\rm h} = G^{\rm h}_0 + G^{\rm h}_0\widetilde{\Sigma}^{\rm h}\widetilde{G}^{\rm h}\,. \tag{25}$$

The BCS pairing mechanism —to be exact, its analog which differs from the phenomenological one by the refinement— has been introduced by means of the known mass operator form

$$\widetilde{\Sigma}_{\rm BCS} = -\widetilde{\Delta}^{(1)}\widetilde{G}^{\rm h}\widetilde{\Delta}^{(2)} \tag{26}$$

and using the Green function

$$\widetilde{G}_{\rm C} = \widetilde{G} + \widetilde{G}\widetilde{\Sigma}_{\rm BCS}\widetilde{G}_{\rm C}\,. \tag{27}$$

Solution of eqs. (24) gives the explicit form of our bare Green functions entering (23)

$$\begin{aligned} &\widetilde{G}_{{\rm C}\lambda}(\varepsilon) = \widetilde{G}^{\rm h}_{{\rm C}\lambda}(-\varepsilon) = \frac{\tilde{u}^2_\lambda}{\varepsilon - \widetilde{E}_\lambda + {\rm i}\delta} + \frac{\tilde{v}^2_\lambda}{\varepsilon + \widetilde{E}_\lambda - {\rm i}\delta}\,, \\ &\widetilde{F}^{(1)}_\lambda(\varepsilon) = -\frac{\widetilde{\Delta}_\lambda}{2\widetilde{E}_\lambda}\left(\frac{1}{\varepsilon - \widetilde{E}_\lambda + {\rm i}\delta} - \frac{1}{\varepsilon + \widetilde{E}_\lambda - {\rm i}\delta}\right)\,, \\ &\widetilde{F}^{(2)}_\lambda(\varepsilon) = \widetilde{F}^{(1)}_\lambda(\varepsilon)\,, \\ &\text{where } \tilde{u}^2_\lambda = 1 - \tilde{v}^2_\lambda = (\widetilde{E}_\lambda + \tilde{\varepsilon}_\lambda)/(2\widetilde{E}_\lambda),\ \widetilde{E}_\lambda = \sqrt{\tilde{\varepsilon}^2_\lambda + \widetilde{\Delta}^2_\lambda}\,. \end{aligned} \tag{28}$$

Thus, specifying the mass operators $M^i$ in eqs. (23), we can obtain useful approximations both for odd and even-even nuclei with pairing. Odd nuclei were discussed in [30], the more difficult case of even ones will be dealt with here. In the following, we omit the index C in all the Green functions.

### 3.2 Equations for the effective fields in non-magic nuclei

The general eqs. (23) for the one-particle Green functions together with similar equations for $G^{\rm h}$ anf $F^{(1)}$ can be written in the matrix form

$$\widehat{G} = \widehat{\widetilde{G}} + \widehat{\widetilde{G}}\widehat{M}\widehat{G}\,, \tag{29}$$

where

$$\begin{aligned} &\widehat{G} = \begin{pmatrix} G & -{\rm i}F^{(1)} \\ {\rm i}F^{(2)} & -G^{\rm h} \end{pmatrix}, \qquad \widehat{\widetilde{G}} = \begin{pmatrix} \widetilde{G} & -{\rm i}\widetilde{F}^{(1)} \\ {\rm i}\widetilde{F}^{(2)} & -\widetilde{G}^{\rm h} \end{pmatrix}, \\ &\widehat{M} = \begin{pmatrix} M & {\rm i}M^{(1)} \\ -{\rm i}M^{(2)} & -M^{\rm h} \end{pmatrix}; \end{aligned} \tag{30}$$

the elements of $\widehat{\widetilde{G}}$ are the Gorkov's Green functions, which have already been refined from the contributions of $\widehat{M}$,

$$\widehat{\widetilde{G}} = \widehat{G}_0 + \widehat{G}_0\widehat{\widetilde{\Sigma}}\widehat{\widetilde{G}}\,. \tag{31}$$

They are known, see eq. (28). The non-diagonal elements of $\widehat{G}_0$ are, of course, zero.

In the Green function approach, the change of the density matrix in an external field $\widehat{V}^0(\omega)$ is connected with the change of the Green functions

$$\hat{\rho}(\omega) \equiv \begin{pmatrix} \rho(\omega) & -{\rm i}\phi^{(1)}(\omega) \\ {\rm i}\phi^{(2)}(\omega) & -\rho^{\rm h}(\omega) \end{pmatrix} = \int \frac{{\rm d}\varepsilon}{2\pi{\rm i}}\delta\widehat{G}(\varepsilon,\omega)\,. \tag{32}$$

Let us obtain a relation for $\delta\widehat{G}$ which will be used in the following. We have from eq. (29)

$$(1 - \widehat{\widetilde{G}}\widehat{M})\delta\widehat{G} = \delta\widehat{\widetilde{G}}(1 + \widehat{M}\widehat{G}) + \widehat{\widetilde{G}}\delta\widehat{M}\widehat{G}\,. \tag{33}$$

Using eq. (29) and the relation $\widehat{G} = (1 - \widehat{\widetilde{G}}\widehat{M})^{-1}\widehat{\widetilde{G}}$, we find

$$\delta\widehat{G} = \widehat{G}(\widehat{V} + \delta\widehat{M})\widehat{G}\,, \tag{34}$$

where the quantity $\widehat{V}$ which determines the change of the Green function $\widetilde{G}$ in the field $\widehat{V}^0$ has been introduced:

$$\delta\widehat{\widetilde{G}} = \widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}}\,. \tag{35}$$

One can obtain from eq. (31) by analogy with the previous derivation

$$\delta\widehat{\widetilde{G}} = \widehat{\widetilde{G}}\widehat{V}^0\widehat{\widetilde{G}} + \widehat{\widetilde{G}}\delta\widehat{\widetilde{\Sigma}}\widehat{\widetilde{G}}\,, \tag{36}$$

*i.e.* the expression for $\widehat{V}$ has the form

$$\widehat{V} = \widehat{V}^0 + \delta\widehat{\widetilde{\Sigma}}\,, \tag{37}$$

where by definition $\delta\widehat{G}_0 = \widehat{G}_0\widehat{V}^0\widehat{G}_0$. We see that the effective field $\widehat{V}$ determines the change of the new mean field and gap in the external field. Like in the case [25] of magic nuclei with account for one-particle–one-hole–phonon configurations, $\widehat{V}$ contains the poles which we need, and the residues of $\widehat{V}$ are connected with the transition probabilities.

In order to obtain a set of manageable final equations, we restrict ourselves to the first order of $M$. This means, though $\widehat{M}$ is not yet specified, we keep in mind that it contains a small parameter. In this order we obtain from eqs. (34,29)

$$\begin{aligned}\delta\widehat{G} \approx (\widehat{\widetilde{G}} + \widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}})(\widehat{V} + \delta\widehat{M})(\widehat{\widetilde{G}} + \widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}}) \approx \\ \widehat{\widetilde{G}}(\widehat{V} + \delta\widehat{M})\widehat{\widetilde{G}} + \widehat{\widetilde{G}}(\widehat{V} + \delta\widehat{M})\widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}} \\ + \widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}}(\widehat{V} + \delta\widehat{M})\widehat{\widetilde{G}} \approx \\ \widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}} + \widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}} + \widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}} + \widehat{\widetilde{G}}\delta\widehat{M}\widehat{\widetilde{G}}\,.\end{aligned} \tag{38}$$

Then the four equations for the effective fields should have the following form:

$$\begin{aligned} V &= V^0 + \bar{\mathcal{U}}\delta G\,, \qquad & d^{(1)} &= d^{01} - \bar{\mathcal{V}}\delta F^{(1)}\,, \\ V^{\rm h} &= V^{0\rm h} + \bar{\mathcal{U}}\delta G^{\rm h}\,, \qquad & d^{(2)} &= d^{02} - \bar{\mathcal{V}}\delta F^{(2)}\,, \end{aligned} \tag{39}$$

where $\delta\widehat{G}$ is determined by eq. (38).

The irreducible amplitudes $\bar{\mathcal{U}}$ and $\bar{\mathcal{V}}$ differ from the corresponding amplitudes $\mathcal{U}$ and $\mathcal{V}$ of the TFFS, because here some contributions to the latter are explicitely singled out and should not be counted twice. It is possible to perform a renormalization procedure like in [17,25]. As a result, we obtain the following system of equations for the effective fields, after substituting eq. (38) into (39):

$$\begin{aligned} V^i &= \bar{e}_q V^{0i} + \bar{F}\{\mathcal{A}^{ij}V^j + \mathcal{A}^{ik}d^k\}\,, \qquad & i,j &= 1,2\,, \\ d^k &= \bar{e}'_q d^{0k} - \bar{F}^\xi\{\mathcal{A}^{ki}V^i + \mathcal{A}^{kl}d^l\}\,, \qquad & k,l &= 3,4\,, \end{aligned} \tag{40}$$

where we have introduced $V^1 = V$, $V^2 = V^{\rm h}$, $d^3 = d^{(1)}$, $d^4 = d^{(2)}$, and the corresponding propagators

$$\mathcal{A}^{ij} = \int K^{ij}\,\frac{{\rm d}\varepsilon}{2\pi{\rm i}}\,. \tag{41}$$

The quantities $K^{ij}$ can be obtained from (38); for the realistic case of the $g^2$ approximation they will be given explicitely in sect. 4. According to eqs. (32), (40), the effective fields are connected with the density matrices, *e.g.*

$$\begin{aligned}\rho(\omega) &= \mathcal{A}^{11}V + \mathcal{A}^{12}V^{\rm h} + \mathcal{A}^{13}d^{(1)} + \mathcal{A}^{14}d^{(2)}\,, \\ \rho^{\rm h}(\omega) &= \mathcal{A}^{21}V + \mathcal{A}^{22}V^{\rm h} + \mathcal{A}^{23}d^{(1)} + \mathcal{A}^{24}d^{(2)}\,.\end{aligned} \tag{42}$$

The equations for $\rho$ and $\rho^{\rm h}$ are

$$\begin{aligned}\rho(\omega) = {} & \mathcal{A}^{11}\bar{e}_q V^{01} + \mathcal{A}^{12}\bar{e}^{\rm h}_q V^{02} \\ & + \mathcal{A}^{13}\bar{e}'^{(1)}_q d^{01} + \mathcal{A}^{14}\bar{e}'^{(2)}_q d^{02} \\ & + \mathcal{A}^{11}\bar{F}\rho + \mathcal{A}^{12}\bar{F}\rho^{\rm h} \\ & + \mathcal{A}^{13}\bar{F}^\xi\phi^{(1)} + \mathcal{A}^{14}\bar{F}^\xi\phi^{(2)}\,, \\ \rho^{\rm h}(\omega) = {} & \mathcal{A}^{21}\bar{e}_q V^{01} + \mathcal{A}^{22}\bar{e}^{\rm h}_q V^{02} \\ & + \mathcal{A}^{23}\bar{e}'^{(1)}_q d^{01} + \mathcal{A}^{24}\bar{e}'^{(2)}_q d^{02} \\ & + \mathcal{A}^{21}\bar{F}\rho + \mathcal{A}^{22}\bar{F}\rho^{\rm h} \\ & + \mathcal{A}^{23}\bar{F}^\xi\phi^{(1)} + \mathcal{A}^{24}\bar{F}^\xi\phi^{(2)}\,.\end{aligned} \tag{43}$$

The quantities $\bar{e}_q$, $\bar{e}^{\rm h}_q$, $\bar{e}'^{(1)}_q$, $\bar{e}'^{(2)}_q$, and $\bar{F}$, $\bar{F}^\xi$ play the role of the local charge $e_q$ and effective interactions $F$, $F^\xi$, respectively, of the TFFS. Strictly speaking they should be determined from comparison with experimental data with the results obtained within the approach under consideration. See, however, subsect. 6.2 below.

### 3.3 Excitation energies and transition probabilities

The excitation energies are determined by solving the system of homogeneous equations which are obtained from eq. (40). The transition probabilities from the ground state to an excited state are found from the renormalized polarization operator (for the case $d^{01} = d^{02} = 0$)

$$\begin{aligned}\langle V^0\rangle = {} & \bar{e}_q V^0\rho + \bar{e}^{\rm h}_q V^{0\rm h}\rho^{\rm h} = \\ & \bar{e}_q V^0(\mathcal{A}^{11}V + \mathcal{A}^{12}V^{\rm h} + \mathcal{A}^{13}d^{(1)} + \mathcal{A}^{14}d^{(2)}) \\ & + \bar{e}^{\rm h}_q V^{0\rm h}(\mathcal{A}^{21}V + \mathcal{A}^{22}V^{\rm h} + \mathcal{A}^{23}d^{(1)} + \mathcal{A}^{24}d^{(2)})\,.\end{aligned} \tag{44}$$

The quantity $\langle V^0\rangle$ determines the strength function which is usually calculated with a smearing parameter

$$S(\omega) = \sum_n |M_{0n}|^2\delta(\omega - E_n) = -\frac{1}{\pi}{\rm Im}\langle V^0\rangle \tag{45}$$

and gives the energy distribution of the excitation strength under consideration. More detailed, *i.e.* nuclear spectroscopy information can be obtained if we calculate the squared amplitude of the transition from the ground to the excited state with energy $E_n$ which is given by

$$|M_{0n}|^2 = \mathrm{Res}|_{\omega=E_n}\langle V^0\rangle\,. \tag{46}$$

This quantity can be expressed in terms of the eigenfunctions which should be calculated from the system of the above-mentioned homogeneous equations. We do not derive this rather involved expression because in practice it is often more convenient to obtain from eq. (45) the quantities $|M_{0n}|^2$ and various sum rules in terms of moments of the strength function $S(\omega)$.

### 3.4 General pairing mass operator. Refinement of the phenomenological single-particle energies and gaps

Let us introduce the new Green functions

$$\begin{aligned}\bar{G} &= G_0 + G_0(\widetilde{\Sigma} + M)\bar{G} = \widetilde{G} + \widetilde{G}M\bar{G}\,,\\ \bar{G}^{\mathrm{h}} &= G_0^{\mathrm{h}} + G_0^{\mathrm{h}}(\widetilde{\Sigma}^{\mathrm{h}} + M^{\mathrm{h}})\bar{G}^{\mathrm{h}} = \widetilde{G}^{\mathrm{h}} + \widetilde{G}^{\mathrm{h}}M^{\mathrm{h}}\bar{G}^{\mathrm{h}}\,,\end{aligned} \tag{47}$$

where $\widetilde{G}$ and $\widetilde{G}^{\mathrm{h}}$ are determined by eqs. (25). Then the system (21) can be transformed to

$$\begin{aligned}G &= \bar{G} - \bar{G}\Sigma^{(1)}F^{(2)}\,,\\ F^{(2)} &= \bar{G}^{\mathrm{h}}\Sigma^{(2)}G\,,\end{aligned} \tag{48}$$

or

$$G = \bar{G} - \bar{G}\Sigma^{(1)}\bar{G}^{\mathrm{h}}\Sigma^{(2)}G \equiv \bar{G} + \bar{G}M_{\mathrm{C}}G\,, \tag{49}$$

where the mass operator

$$M_{\mathrm{C}} = -\Sigma^{(1)}\bar{G}^{\mathrm{h}}\Sigma^{(2)} \tag{50}$$

is a generalization of eq. (6), or (26) for the "refined" pairing. The formula (50) is a general expression for the mass operator which is responsible for pairing in a Fermi system.

It is easy to obtain another form for eq. (49):

$$G = \widetilde{G} + \widetilde{G}(M + M_{\mathrm{C}})G\,. \tag{51}$$

Using the results of this section one can obtain a generalization to the quasiparticle-phonon interaction and pairing case of the results of the TFFS dealing with the connection of single-particle energies and the mass operator. For simplicity and bearing in mind further applications we will consider all the quantities in eq. (49) in diagonal approximation in the single-particle index $\lambda$

$$G_\lambda = \bar{G}_\lambda - \bar{G}\Sigma_\lambda^{(1)}\bar{G}_\lambda^{\mathrm{h}}\Sigma_\lambda^{(2)}G_\lambda\,. \tag{52}$$

Following the idea of [31] let us represent the mass operators $M$ and $M^{\mathrm{h}}$ as sums of odd and even parts, *e.g.*,

$$M = M_{\mathrm{ev}} + M_{\mathrm{odd}}\,. \tag{53}$$

Then defining the energies which we need as poles of the Green function $G_\lambda$ one can obtain from eq. (52) the formal general expression for these energies $E_{\lambda\eta}$ in the known form

$$E_{\lambda\eta} = \sqrt{\varepsilon_{\lambda\eta}^2 + \Delta_{\lambda\eta}^2}\,, \tag{54}$$

where

$$\varepsilon_{\lambda\eta} = \frac{\tilde{\varepsilon}_\lambda + M_{\mathrm{ev}\lambda}(E_{\lambda\eta})}{1 + q_{\lambda\eta}}\,, \quad \Delta_{\lambda\eta}^2 = \frac{\Sigma_\lambda^{(1)}(E_{\lambda\eta})\Sigma_\lambda^{(2)}(E_{\lambda\eta})}{(1+q_{\lambda\eta})^2} \tag{55}$$

and $q_{\lambda\eta} = -M_{\mathrm{odd}\lambda}(E_{\lambda\eta})/E_{\lambda\eta}$. The index $\eta$ enumerates the solutions of the system (54)-(55).

Now it is possible to obtain the formulae for the connection between the phenomenological (or observed) quantities $\{\varepsilon_\lambda,\ \Delta_\lambda\}$ and the refined ones $\{\tilde{\varepsilon}_\lambda,\ \widetilde{\Delta}_\lambda\}$. The experimental single-particle energies should correspond to dominant levels, *i.e.* the levels which have the largest spectroscopic factors. Therefore, the refinement should be done in such a manner that one of the solutions should coincide with the experimental value, and this level should be the dominant one.

These experimental single-particle energies are initial data for the whole problem. We denote these energies and the corresponding gaps as $\varepsilon_\lambda$ and $\Delta_\lambda$. Then using this condition and also eqs. (22) and (55), we obtain the general relations between observed and refined quantities

$$\begin{aligned}\varepsilon_\lambda &= \frac{\tilde{\varepsilon}_\lambda + M_{\mathrm{ev}\lambda}(E_\lambda)}{1 + q_\lambda(E_\lambda)}\,,\\ \Delta_\lambda \equiv \Delta_\lambda^{(1)} &= \Delta_\lambda^{(2)} = \frac{\widetilde{\Delta}_\lambda^{(2)} + M_\lambda^{(2)}(E_\lambda)}{1 + q_\lambda(E_\lambda)}\,,\\ E_\lambda &= \sqrt{\varepsilon_\lambda^2 + \Delta_\lambda^2}\,,\end{aligned} \tag{56}$$

with $q_\lambda = -M_{\mathrm{odd}\lambda}(E_\lambda)/E_\lambda$. This leads in the case $\Delta_\lambda = 0$ to the prescription $\varepsilon_\lambda = \tilde{\varepsilon}_\lambda + M_\lambda(\varepsilon_\lambda)$ which has been used earlier for magic nuclei [5,25].

## 4 The $g^2$ approximation

In our approach, the specific features of non-magic nuclei are connected also with the new quantities $M^{(1)}$ and $M^{(2)}$ which were absent in magic nuclei. To analyze them in a transparent way and to present a numerically simple example of the new approach it is of interest to consider the case of the $g^2$ approximation (where $g$ is the particle-phonon coupling). We mean by this the restriction to the case when the elements $M^i$ of $\widehat{M}$ contain only $g^2$ terms, and Green functions $\widehat{\widetilde{G}}$ instead of the total Green

functions $\widehat{G}$. This case is the realistic one for semi-magic nuclei. In [30] the corresponding dimensionless small parameters in $^{120}$Sn have been calculated for the low-lying $2^+$ and $3^-$ levels, which give the largest contributions to the two-quasiparticle–phonon configurations. It has been found that this parameter is less than one.

In this section we specialize the formulae to the $g^2$ approximation.

### 4.1 Additional approximations

Even in the $g^2$ approximation, the formulae obtained will be rather involved, due to the necessity to account for the particle-particle and hole-hole interaction both in eq. (40) and in the system of the TFFS equations (19) (the latter is necessary to calculate the phonons used for the two-quasiparticle–phonon configurations). Therefore, in order to get explicit and still relatively short expressions, we omit the particle-particle and hole-hole channels in both cases. It is easy to abandon this approximation, if necessary. This necessity might arise, *e.g.* when checking the numerical performance of conservation laws; but we expect that the numerical contributions of these channels are small enough for excited states [17,32].

Our first approximation (which is still on the QRPA level) consists of setting $d^{(1)} = d^{(2)} = 0$ in the set of eqs. (19), and of neglecting pairing phonons in all our $M^i$ taken in the $g^2$ approximation. It is easily seen using the formula $\widehat{M} = \hat{g}\mathcal{D}\widehat{\widetilde{G}}\hat{g}$ that this approximation, $g^{(1)} = g^{(2)} = 0$, gives only one term instead of four for each $M^i$. Because we will use only low-lying and collective phonons in our two-quasiparticle–phonon configurations, the corresponding corrections to these from the pairing phonons should be small [32].

We thus obtain the following form of the mass operators

$$M_1(\varepsilon) = M_1^{\rm h}(-\varepsilon) = \sum_{2,s}(g_{12}^s)^2\left\{\frac{\tilde{u}_2^2}{\varepsilon-\widetilde{E}_2-\omega_s+{\rm i}\delta}+\frac{\tilde{v}_2^2}{\varepsilon+\widetilde{E}_2+\omega_s-{\rm i}\delta}\right\},$$
$$M_1^{(1)}(\varepsilon) = M_1^{(2)}(\varepsilon) = -\sum_{2,s}(g_{12}^s)^2\frac{\widetilde{\Delta}_2}{2\widetilde{E}_2}\left\{\frac{1}{\varepsilon-\widetilde{E}_2-\omega_s+{\rm i}\delta}-\frac{1}{\varepsilon+\widetilde{E}_2+\omega_s-{\rm i}\delta}\right\}. \quad (57)$$

Here the lower index $1 \equiv \{n_1, l_1, j_1, m_1\}$ (for spherical nuclei), $\widetilde{E}_2 = \sqrt{\tilde{\varepsilon}_2^2 + \widetilde{\Delta}_2^2}$. One can see from eq. (57) that in the case of $\widetilde{\Delta} = 0$, there is no contribution of $M^{(1)}$ and $M^{(2)}$.

Our second additional approximation is $d^{(1)} = d^{(2)} = 0$ in eqs. (40), (42), (44). This also can be abandoned in analogy with the results below, if experimental data should call for it.

### 4.2 Derivation of the basic equations

Here the general equations of sect. 3 will be briefly described using the $g^2$ approximation and the additional approximations of the preceding subsect. 4.1. The results will be given in symbolic notation, while the explicit form is given in the Appendix.

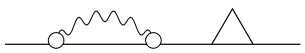
#### 4.2.1 The effective fields and density matrices

The explicit form of $\delta\widehat{G}$ of eq. (38) is needed. The first term $\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}}$, when substituted into eqs. (39) and (40), gives equations which are similar to (19) of the TFFS but contain refined Green functions and, therefore, $\tilde{\varepsilon}_\lambda$ and $\widetilde{\Delta}_\lambda$. In our approximations, $\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}}$ is given by

$$\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}} = \delta\widehat{\widetilde{G}} = \begin{pmatrix} \delta\widetilde{G} & -{\rm i}\delta\widetilde{F}^{(1)} \\ {\rm i}\delta\widetilde{F}^{(2)} & -\delta\widetilde{G}^{\rm h}\end{pmatrix} = \begin{pmatrix} \widetilde{G}V\widetilde{G}-\widetilde{F}^{(1)}V^{\rm h}\widetilde{F}^{(2)} & -{\rm i}\widetilde{G}V\widetilde{F}^{(1)}-{\rm i}\widetilde{F}^{(1)}V^{\rm h}\widetilde{G}^{\rm h} \\ {\rm i}\widetilde{F}^{(2)}V\widetilde{G}+{\rm i}\widetilde{G}^{\rm h}V^{\rm h}\widetilde{F}^{(2)} & \widetilde{F}^{(2)}V\widetilde{F}^{(1)}-G^{\rm h}V^{\rm h}G^{\rm h}\end{pmatrix}. \quad (58)$$

We call the terms $\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}}$ and $\widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}}$ graphs with right and left self-energy insertion, respectively, describing the place of $\widehat{M}$ with respect to $\widehat{V}$ in the above expressions. The term $\widehat{\widetilde{G}}\delta\widehat{M}\widehat{\widetilde{G}}$ corresponds to graphs where a phonon is exchanged in the crossed channel (phonon exchange graphs).

According to our additional approximations, we should also put $d^{(1)} = d^{(2)} = 0$ in the matrix $\widehat{V}$ and in eqs. (40), so that for this system we have only the quantities $\mathcal{A}^{11}$, $\mathcal{A}^{12}$, $\mathcal{A}^{21}$, and $\mathcal{A}^{22}$. Let us obtain, as an example, the formulae for the "left" insertion graphs

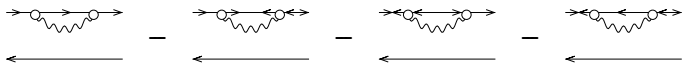


$$\longrightarrow \quad \widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}} =$$
$$\begin{pmatrix} \widetilde{G} & -{\rm i}\widetilde{F}^{(1)} \\ {\rm i}\widetilde{F}^{(2)} & -\widetilde{G}^{\rm h}\end{pmatrix}\begin{pmatrix} M & {\rm i}M^{(1)} \\ -{\rm i}M^{(2)} & -M^{\rm h}\end{pmatrix}$$
$$\times\begin{pmatrix} \widetilde{G} & -{\rm i}\widetilde{F}^{(1)} \\ {\rm i}\widetilde{F}^{(2)} & -\widetilde{G}^{\rm h}\end{pmatrix}\begin{pmatrix} V & 0 \\ 0 & -V^{\rm h}\end{pmatrix}\begin{pmatrix} \widetilde{G} & -{\rm i}\widetilde{F}^{(1)} \\ {\rm i}\widetilde{F}^{(2)} & -\widetilde{G}^{\rm h}\end{pmatrix}. \quad (59)$$

For the matrix element $(\widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}})_{11}$ it is easy to obtain

$$(\widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}})_{11} = (\widetilde{G}M\widetilde{G} - \widetilde{G}M^{(1)}\widetilde{F}^{(2)} - \widetilde{F}^{(1)}M^{(2)}\widetilde{G} - \widetilde{F}^{(1)}M^{\rm h}\widetilde{F}^{(2)})V\widetilde{G} + (-\widetilde{G}M\widetilde{F}^{(1)} - \widetilde{G}M^{(1)}\widetilde{G}^{\rm h} + \widetilde{F}^{(1)}M^{(2)}\widetilde{F}^{(1)} - \widetilde{F}^{(1)}M^{\rm h}\widetilde{G}^{\rm h})V^{\rm h}\widetilde{F}^{(2)}, \quad (60)$$

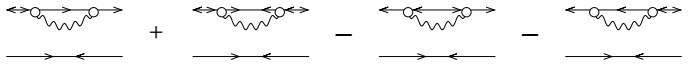
so that the quantities $K_{\rm left}^{11}$ and $K_{\rm left}^{12}$ appearing in eq. (41) are given by

$K^{11}_{\rm left} =$  ,

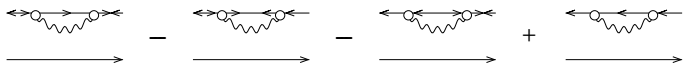
$K^{12}_{\rm left} =$  . (61)

Analogously one can obtain the matrix element $(\widehat{\widetilde{G}}\widehat{M}\widehat{\widetilde{G}}\widehat{V}\widehat{\widetilde{G}})_{22}$ and the corresponding quantities $K^{21}_{\rm left}$ and $K^{22}_{\rm left}$

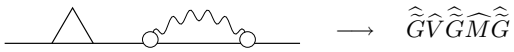
$K^{21}_{\rm left} = (-\widetilde{F}^{(2)}M\widetilde{G} + \widetilde{F}^{(2)}M^{(1)}\widetilde{F}^{(2)} - \widetilde{G}^{\rm h}M^{(2)}\widetilde{G} - \widetilde{G}^{\rm h}M^{\rm h}\widetilde{F}^{(2)})\widetilde{F}^{(1)} =$

 ,

$K^{22}_{\rm left} = (-\widetilde{F}^{(2)}M\widetilde{F}^{(1)} - \widetilde{F}^{(2)}M^{(1)}\widetilde{G}^{\rm h} - \widetilde{G}^{\rm h}M^{(2)}\widetilde{F}^{(1)} + \widetilde{G}^{\rm h}M^{\rm h}\widetilde{G}^{\rm h})\widetilde{G}^{\rm h} =$

 . (62)

Similarly the "right" insertion graphs can be obtained

 (63)

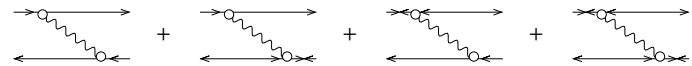
in our approximations. The corresponding formulae for the $\mathcal{A}^{ij}$ $(i,j=1,2)$ are given in the Appendix.
Next consider the phonon exchange contribution. In the $g^2$ approximation the variation of $\widehat{M}$ in the external field is

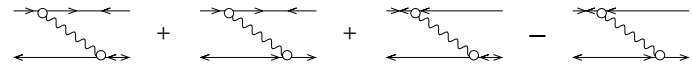
$$\delta\widehat{M} = gD\delta\widehat{\widetilde{G}}g\,. \tag{64}$$

Variations of $g$ and $D$ do not enter because $\delta g$ and $\delta D$ are of higher order of smallness than $g^2$ [25]. Substituting eq. (58) into (64), the formulae for the matrix elements of $\widehat{\widetilde{G}}\delta\widehat{M}\widehat{\widetilde{G}}$ are obtained, *e.g.*,

$$\begin{aligned}(\widehat{\widetilde{G}}\delta\widehat{M}\widehat{\widetilde{G}})_{11} = {} & (\widetilde{G}\widetilde{G}gDg\widetilde{G}\widetilde{G} + \widetilde{G}\widetilde{G}gDgF^{(1)}\widetilde{F}^{(2)} + \widetilde{F}^{(1)}\widetilde{F}^{(2)}gDg\widetilde{G}\widetilde{G} + \widetilde{F}^{(1)}\widetilde{F}^{(2)}gDgF^{(1)}\widetilde{F}^{(2)})V \\ & + (-\widetilde{G}\widetilde{F}^{(1)}gDgF^{(2)}\widetilde{G} + \widetilde{G}\widetilde{F}^{(1)}gDg\widetilde{G}^{\rm h}\widetilde{F}^{(2)} + \widetilde{F}^{(1)}\widetilde{G}^{\rm h}gDgF^{(2)}\widetilde{G} - \widetilde{F}^{(1)}\widetilde{G}^{\rm h}gDg\widetilde{G}^{\rm h}\widetilde{F}^{(2)})V^{\rm h},\end{aligned} \tag{65}$$

so that the quantities $K^{11}_{\rm exch}$ and $K^{12}_{\rm exch}$ which give the contributions to $K^{11}$ and $K^{12}$ caused by the cross phonon graphs, are given by

$K^{11}_{\rm exch} =$  ,

$K^{12}_{\rm exch} =$  . (66)

An analogous treatment of $(\widehat{\widetilde{G}}\delta\widehat{M}\widehat{\widetilde{G}})_{22}$ yields $K^{21}_{\rm exch}$ and $K^{22}_{\rm exch}$.

The expressions for the total quantities

$$\mathcal{A}^{ik}_{1234s}(\omega) = \int (K^{ik}_{\rm left} + K^{ik}_{\rm right} + K^{ik}_{\rm exch})_{1234s}(\varepsilon,\omega)\frac{{\rm d}\varepsilon}{2\pi{\rm i}} \tag{67}$$

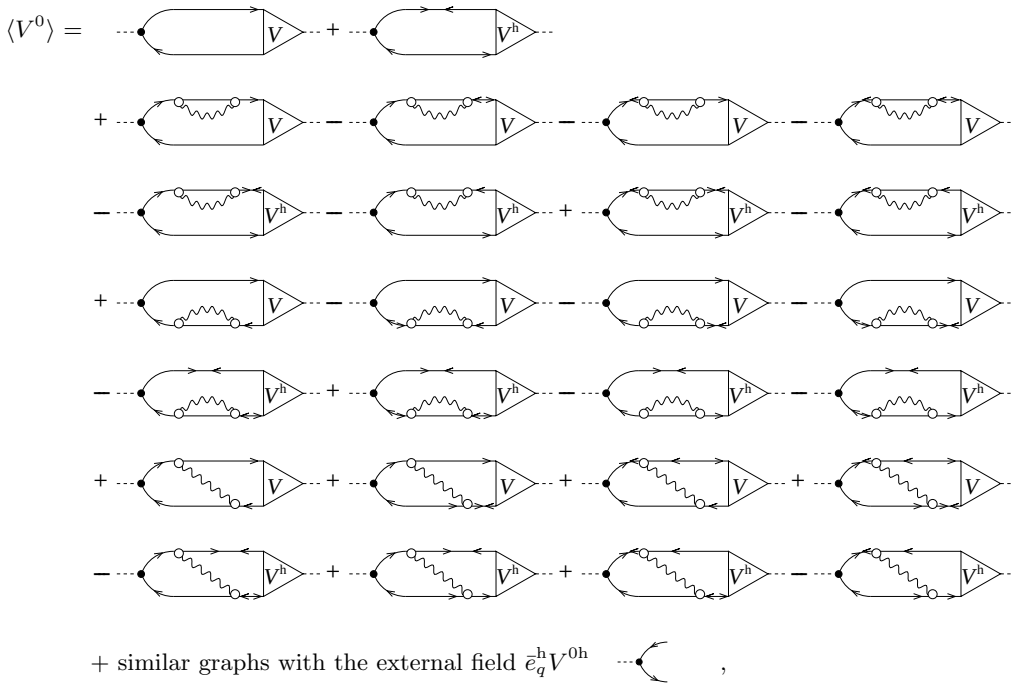
can be obtained using the Appendix.

Now we are able to write down the complete formulae for $\delta G$ and $\delta G^{\rm h}$ in our approximations:

$$\begin{aligned}\delta G &= K^{11}V + K^{12}V^{\rm h}\,,\\ \delta G^{\rm h} &= K^{21}V + K^{22}V^{\rm h}\,.\end{aligned} \tag{68}$$

The quantities $K^{ik}$ are given in the Appendix. The equations for the effective fields $V$ and $V^{\rm h}$ in our approximations are given by

$$\begin{aligned}V &= \bar{e}_q V^0 + \bar{F}\{\mathcal{A}^{11}V + \mathcal{A}^{12}V^{\rm h}\}\,,\\ V^{\rm h} &= \bar{e}^{\rm h}_q V^{0{\rm h}} + \bar{F}\{\mathcal{A}^{21}V + \mathcal{A}^{22}V^{\rm h}\}\,.\end{aligned} \tag{69}$$




(72)

According to eqs. (32), (68) or (42), the density matrices $\rho$, $\rho^{\rm h}$ are connected with the effective fields as follows:

$$\rho(\omega) = \mathcal{A}^{11} V + \mathcal{A}^{12} V^{\rm h}\,,$$
$$\rho^{\rm h}(\omega) = \mathcal{A}^{21} V + \mathcal{A}^{22} V^{\rm h}\,, \qquad (70)$$

and the equations for them are

$$\rho(\omega) = \mathcal{A}^{11}\bar{e}_q V^0 + \mathcal{A}^{12}\bar{e}_q^{\rm h} V^{0\rm h} + \mathcal{A}^{11}\bar{F}\rho + \mathcal{A}^{12}\bar{F}\rho^{\rm h}\,,$$
$$\rho^{\rm h}(\omega) = \mathcal{A}^{21}\bar{e}_q V^0 + \mathcal{A}^{22}\bar{e}_q^{\rm h} V^{0\rm h} + \mathcal{A}^{21}\bar{F}\rho + \mathcal{A}^{12}\bar{F}\rho^{\rm h}\,. \qquad (71)$$

#### 4.2.2 The polarization operator

According to eq. (44) the polarization operator in our approximations is given by the following graphs:

*see eq.* (72) *above*,

where the thick point indicates that in the medium the local charges $\bar{e}_q$ and $\bar{e}_q^{\rm h}$ enter.

## 5 Inclusion of the single-particle continuum

Taking into account the continuum is important for several reasons. Part of them was considered in ref. [33] on the QRPA level, we will discuss here the role of the continuum in connection with the complex configurations' problem.

1. The continuum gives the real physical envelope for the processes at excitation energies higher than the nucleon separation energy. For the giant resonance, this gives the escape width $\Gamma^{\uparrow}$. The contribution of it depends, in general, noticeably on the atomic number, the excitation energy etc. So it is not very realistic, especially in calculations of such delicate properties as fine structure and decay characteristics, to imitate the role of continuum by a constant smearing parameter. The realistic treatment of the continuum is important for $E1$ giant resonances in $^{40}$Ca and $^{48}$Ca [10] and, of course, in light nuclei.
   As is well known it is possible to include the single-particle continuum exactly, *i.e.* to avoid any truncation of the $\lambda$-basis, within the RPA approach in the coordinate representation [34]. A generalization of this approach to the QRPA has been done in [33] for the description of the first $2^+$ levels where the method of the so-called mixed $\{\vec{r},\lambda\}$-representation has been used.
2. In the complex configurations problem under consideration, using the $\lambda$-representation gives matrices of a very large rank especially in the case of treatment of the ground-state correlations caused by complex configurations. Using the coordinate representation has a big numerical advantage in this problem because the rank of matrices is now determined not by the number of configurations but by the number of mesh points where the corresponding integral equation is solved.
3. Last not least, it should be emphasized also that only a reliable inclusion of the single-particle continuum makes it possible to calculate nuclei with nucleon separation energy near zero. This is important for drip line nuclei and for astrophysical studies.

Thus, we generalize the approach for magic nuclei of ref. [10] to include pairing.

In coordinate representation eqs. (71) have the form

$$\begin{aligned}\rho(\vec{r},\omega) &= \int \mathcal{A}^{11}(\vec{r},\vec{r}',\omega)\bar{e}_q V^0(\vec{r}')\mathrm{d}\vec{r}' \\ &+ \int \mathcal{A}^{12}(\vec{r},\vec{r}',\omega)\bar{e}_q^{\mathrm{h}} V^{0\mathrm{h}}(\vec{r}')\mathrm{d}\vec{r}' \\ &+ \int \mathcal{A}^{11}(\vec{r},\vec{r}',\omega)\bar{F}(\vec{r}')\rho(\vec{r}',\omega)\mathrm{d}\vec{r}' \\ &+ \int \mathcal{A}^{12}(\vec{r},\vec{r}',\omega)\bar{F}(\vec{r}')\rho^{\mathrm{h}}(\vec{r}',\omega)\mathrm{d}\vec{r}' \,, \\ \rho^{\mathrm{h}}(\vec{r},\omega) &= \int \mathcal{A}^{21}(\vec{r},\vec{r}',\omega)\bar{e}_q V^0(\vec{r}')\mathrm{d}\vec{r}' \\ &+ \int \mathcal{A}^{22}(\vec{r},\vec{r}',\omega)\bar{e}_q^{\mathrm{h}} V^{0\mathrm{h}}(\vec{r}')\mathrm{d}\vec{r}' \\ &+ \int \mathcal{A}^{21}(\vec{r},\vec{r}',\omega)\bar{F}(\vec{r}')\rho(\vec{r}',\omega)\mathrm{d}\vec{r}' \\ &+ \int \mathcal{A}^{22}(\vec{r},\vec{r}',\omega)\bar{F}(\vec{r}')\rho^{\mathrm{h}}(\vec{r}',\omega)\mathrm{d}\vec{r}', \end{aligned} \tag{73}$$

where we have specialized to an effective contact force $\bar{F}(\vec{r})$, see subsect. 6.2 below, and the propagators are given by

$$\begin{aligned}\mathcal{A}^{11}(\vec{r},\vec{r}',\omega) &= \sum_{1234} \mathcal{A}^{11}_{1234}\phi_1^*(\vec{r})\phi_2(\vec{r})\phi_3(\vec{r}')\phi_4^*(\vec{r}') \,, \\ \mathcal{A}^{12}(\vec{r},\vec{r}',\omega) &= \sum_{1234} \mathcal{A}^{12}_{1234}\phi_1^*(\vec{r})\phi_2(\vec{r})\phi_3(\vec{r}')\phi_4^*(\vec{r}') \end{aligned} \tag{74}$$

and so forth. To calculate the propagators in the coordinate representation in our problem with pairing it is convenient to use the fact that pairing is important only for a small number of levels near the Fermi surface. Thus, since the propagators $A^{12}$ and $A^{21}$ are proportional to $\tilde{\Delta}^2$ (see Appendix), we use eq. (74) for them and for the propagator $A^{11}$ we take

$$\begin{aligned}&\mathcal{A}^{11}(\vec{r},\vec{r}',\omega) = \tilde{\mathcal{A}}^{\mathrm{QRPA}}(\vec{r},\vec{r}',\omega) \\ &+ \sum_{1234}(\mathcal{A}^{11}_{1234} - \tilde{\mathcal{A}}^{\mathrm{QRPA}}_{1234}\delta_{13}\delta_{24})\tilde{\phi}_1^*(\vec{r})\tilde{\phi}_2(\vec{r})\tilde{\phi}_3(\vec{r}')\tilde{\phi}_4^*(\vec{r}') \end{aligned} \tag{75}$$

and similary for $\mathcal{A}^{22}$. Here $\widetilde{\mathcal{A}}^{\mathrm{QRPA}}$ is the corresponding "refined" QRPA propagator in which the single-particle spectrum is taken into account exactly within the RPA and the corrections caused by pairing are calculated explicitly, for example

$$F^{(1,2)}(\vec{r},\vec{r}',\epsilon) = -\sum_1 \frac{\tilde{\Delta}_1}{\epsilon^2 - \tilde{E}_1^2}\tilde{\phi}_1^*(\vec{r})\tilde{\phi}_1(\vec{r}') \tag{76}$$

and for the part $\delta G$ of the Green function responsible for pairing

$$\delta G(\vec{r},\vec{r}',\epsilon) = \sum_1 \frac{\tilde{\Delta}_1^2}{(\epsilon - \tilde{\epsilon}_1)(\epsilon^2 - \tilde{E}_1^2)}\tilde{\phi}_1^*(\vec{r})\tilde{\phi}_1(\vec{r}') \tag{77}$$

(for details see refs. [5,10,33]). Like in refs. [5,10] the summation in eq. (75) should be performed in a basis which is not less than two shells above and two below the Fermi surface plus the unfilled shell near it.

Such a combined method is most convenient for numerical calculations. Making use of eq. (75) means that we underestimate the contribution of the single-particle continuum. The calculations in magic nuclei [5,10], where a formula similar to (75) was used, have shown that this underestimation is not large for giant resonances.

## 6 Discussion of the main equations

Here we will discuss some pecularities and the physical approximations of the approach.

### 6.1 Ground-state correlations due to complex configurations

In the Hartree-Fock approximation, the nuclear ground state is assumed to be that of a gas of non-interacting fermions in a potential well. The presence of the residual interaction changes this state to what is called a correlated ground state. The ground-state correlations are treated in different approximations; most important and best known are the pairing correlations and the RPA ground-state correlations.

In a graphical expansion, the presence of ground-state correlations manifests itself in the possibility of (sub-)graphs which correspond to the creation of a number of virtual elementary excitations (particles, holes, quasiparticles, or phonons) from the vacuum, or the opposite process of annihilation leading to the vacuum state again. In the case of pairing correlations, this is the spontaneous creation of a correlated particle-particle pair (which emerge from a kind of condensate), while RPA-type correlations correspond to the elementary process of creation of two particle-hole pairs from the vacuum by the residual interaction (which, of course, may happen more than once, yielding components with four, six, ... particle-hole pairs).

In the case of pairing, the Bogolyubov-quasiparticles are determined in such a way that the spontaneous generation of a quasiparticle pair is impossible, while the quasiparticle-RPA ground-state correlations are due the possibility of the elementary process of spontaneous generation of four quasiparticles.

Our present approach incorporates all the correlations mentioned so far, but in addition also terms which are not included on the (Q)RPA level. We will denote the latter by $\mathrm{GSC}_{\mathrm{phon}}$. There may be cases when only the diagrams corresponding to $\mathrm{GSC}_{\mathrm{phon}}$ exist, *i.e.* all the effect is determined by $\mathrm{GSC}_{\mathrm{phon}}$, as noted in the introduction. Thus, these new $\mathrm{GSC}_{\mathrm{phon}}$ lead to a redistribution of the strength or sometimes to the appearance of new transitions in which the external field couples the correlated ground state directly to the two-quasiparticle–phonon configurations. In this latter sense our approach is qualitatively different from the RPA or QRPA.

For magic nuclei ground-state correlations which are similar to our $GSC_{phon}$ have been calculated and discussed in refs. [5,13,14,16]. In ref. [5] there is also a detailed description of the pole and diagram structure of the ground-state correlations which were considered there. In particular, it was obtained that they give an increase of the energy weighted sum rules of 4–7% for the isovector $E1$ and isoscalar $E0$, $E2$ resonances in $^{40}$Ca, $^{48}$Ca and $^{56}$Ni. Taking any ground-state correlations beyond the (Q)RPA into account is very involved numerically and making use of the coordinate representation decreases the difficulties strongly, as discussed in sect. 5.

One can expect that $GSC_{phon}$ may be quantitatively important at least for low-lying excitations in non-magic nuclei.

### 6.2 The effective interactions and local charges

In all our equations the new effective interactions $\bar{F}, \bar{F}^{\xi}$ and local charges $\bar{e}_q, \bar{e}'_q$ enter. In general, their parameters should be determined in the framework of the present approach and these quantities should differ from the old ones because we have singled out explicitely the most "dangerous" complex configurations from $F$, $e_q$ and $F^{\xi}, e'_q$.

However, as in the case of magic nuclei, we are going to use a relatively small number of collective and low-lying phonons which give the main contribution to the characteristics of giant resonances and the low-lying excitation spectrum. (A more detailed discussion can be found in ref. [12].)

The corrections of the effective interactions and local charges due to the inclusion of the phonons under consideration must be irregular and of a long-range character. Thus, one can hope that these *non-local* corrections of the *local* quantities under discussion are small and we can use the old parameters of $F$ and $e_q$ instead of the new ones at least as a first approximation. This approximation has been confirmed by the calculations in magic nuclei [5, 10–16]. A similar conclusion about $F^{\xi}$ and $\bar{F}^{\xi}$ is also confirmed in our preliminary results of solutions of the gap equations containing $F^{\xi}$ and $\bar{F}^{\xi}$.

### 6.3 The "smearing" parameter

The main results of our approach have been formulated in sects. 3 and 4 in the discrete $\tilde{\lambda}$ representation (to be exact: in the representation of the wave function $\tilde{\phi}_{\lambda}$) and also in sect. 5 in the coordinate representation. In both cases it is necessary to introduce a smearing parameter $\eta$ which roughly describes the contribution of those complex configurations which were not taken into account explicitely and, in addition, simulates the finite experimental resolution. So we choose $\eta$ to be equal, at least approximately, to the experimental resolution. In the calculations of giant resonances in magic nuclei [5,10–14,16] we used the smearing parameter $\eta = \Gamma/2$ ($\Gamma$ is the width of the Breit-Wigner distribution for an isolated peak of the strength function) from the range $\eta = 100$–$500$ keV. However, when studying the fine structure, *e.g.*, for the isoscalar $E2$ resonance in $^{208}$Pb [15], we used the parameter $\eta = 20$ keV and obtained a reasonable description of the experiment (see also the discussion on the resonance fine structure and experimental resolution in ref. [13]). The use of the smearing parameter decreases the numerical difficulties considerably.

It is clear that, in general, the smearing parameter should depend on the excitation energy and it would be desirable to have a more realistic description of the appropriate effects. Taking into account the known difficulties with a full microscopic description of the optical potential there is a sense to use the known phenomenological "refined" optical potential. This procedure will be described elsewhere.

### 6.4 Other approximations

In the formulae for the mass operators (1), (2) and for the propagators we use the phonons calculated separately and microscopically within the TFFS. Therefore, as it is known for the case of magic nuclei, there is double counting of the lowest-order contribution to the corresponding mass operators. However, since only collective phonons are accounted for, the error introduced by this should be negligible. That was also shown numerically earlier in ref. [35]

In addition, using these, in fact, phenomenological phonons means that they contain some additional contributions which should be excluded, strictly speaking, in the famework of our approach. However, the corresponding contributions (refinement of phonons) are of order $g^4$ and need not to be considered here.

It is necessary to emphasize that we mean everywhere the relatively small number of phonons which are selected in such a way that the energies of the appropriate complex configurations are roughly in the energy region under consideration. All the rest of the configurations in the mass operators are effectively accounted for by the use of refined single-particle energies $\tilde{\epsilon}_{\lambda}$, gaps $\tilde{\Delta}_{\lambda}$ and the parameter $\eta$. Thus, our assumption is that the rest of the configurations in the mass operators can be taken into account on a simple phenomenological level. This is an extension and application of the ideas of the standard TFFS to a phenomenological treatment of the configurations which are more complex than the RPA ones by the introduction of effective interactions and local charges. Due to such an approach we avoid, at least on the present level of understanding of the problem, the difficult questions of explicit treatment of all the complex configurations and of the full microscopic description of the optical potential. On the other hand, our refinement procedure for single-particle energies and gaps (subsect. 3.4) is satisfactory numerically because in our mass operators we take into account only the same phonons which are explicitely accounted for in our final formulae.

During the numerical realization of the approach in the $g^2$ approximation discussed (sect. 4) difficulties connected with unphysical second-order poles of the propagators may appear. As it has been shown in ref. [22], for

the isovector $M1$ resonances in magic nuclei these poles are at the very lowest part of the energy spectrum and contribute to the sum rules only negligibly. The same situation is expected in non-magic even-even nuclei, at least for the isovector excitations. However, it is desirable to implement a procedure to circumvent these difficulties.

To each of the second-order poles one can find higher-order graphs corresponding to third-order, fourth-order ... poles. Together with the diagram yielding the first-order pole, this subclass of graphs can be summed as a geometrical series, with the result of a sum of only first-order pole terms [36,5,10]. The best known example for this is the solution of the Dyson equation for the single-particle Green function which sums up any number of insertions of the mass operator into the unperturbed propagator. Similar procedures work also for two-particle propagators, or more complicated graphs in general. An example is presented in sect. 7.

Though in such a procedure graphs of arbitrary order in $g^2$ are summed up, one should be aware of the fact that already in order $g^4$, some terms are missing, and the relative number of omitted terms increases with increasing order of the graphs. Thus, by doing such partial summation, one is not really going beyond the $g^2$ approximation, since already the order $g^4$ is not correctly included. Therefore, in special cases, simpler approximate methods to eliminate unphysical poles may be preferrable.

For magic nuclei, an approximate procedure of summation of the $g^2$ terms in the propagator has been developed in [5,10]. But for non-magic nuclei such a summation would be too involved and hardly feasible at present. The development of a simpler procedure is in progress.

## 7 Specific cases and schematic models

It would be interesting to analyse more in detail a simplified variant of our problem which is rather complicated algebraically and numerically, especially for realistic non-separable forces. The equations obtained describe the influence of phonon coupling or, more precisely, particle-vibration coupling (complex configurations), on processes described within the QRPA (simple configurations). Therefore, a corresponding generalization of the Brown-Bolsterli model for separable forces [37,38] would be of interest. To our knowledge, no direct generalization has ever been performed for non-magic nuclei. This will be done in the present section in a two-level model.

We do not expect principal changes due to taking $\mathrm{GSC_{phon}}$ into account, at least as compared with those given by the complex configurations on the whole (except several cases, see [13,22]). On the other hand, they will seriously complicate the problem, as mentioned above. For these reasons, we will not include them into our analysis. To be exact, we omit the fermion $\mathrm{GSC_{phon}}$ only, but the $\mathrm{GSC_{phon}}$ caused by the non-pole term of the Green function (GF) $D_s$ in the cross phonon graphs will be included. This is done mainly in order to compare with another method of accounting for complex configurations in magic nuclei based on Nuclear Field Theory [2] where the latter kind of the $\mathrm{GSC_{phon}}$ was included. However, an analysis of $\mathrm{GSC_{phon}}$ turned out to be possible in the frame of a one-level model for non-magic nuclei, see at the end of subsect. 7.1 below.

The main simplification of the model is the use of separable forces, *i.e.* in eq. (69) we take

$$\bar{F}_{1234} = \lambda q_{12} q_{34}. \tag{78}$$

Then for the problem with a propagator $\mathcal{A}^{(11)}_{121'2'}$, which contains both simple and complex configurations, the creation amplitude of an observed phonon with energy $\omega_n$ which satisfies the equation

$$g^n_{34} = \sum_{121'2'} \bar{F}_{3412} \mathcal{A}^{(11)}_{121'2'}(\omega_n) g^n_{1'2'} \tag{79}$$

is given by $g^n_{12} = C(\omega_n) q_{12}$ and eq. (79) gives the secular equation:

$$\frac{1}{\lambda} = \sum_{121'2'} q_{12} \mathcal{A}^{(11)}_{121'2'}(\omega_n) q_{1'2'}. \tag{80}$$

eqs. (79), (80) describe magic nuclei if we take $\tilde{\Delta} = 0$ in the Appendix. For the RPA case in eq. (80) one should take $\mathcal{A}^{(11)}_{121'2'} = \mathcal{A}^{11;0}_{121'2'}$.

### 7.1 Specific cases

If one accounts only for simple configurations without GSC, then the propagator can be obtained from the RPA case by keeping only one of two terms of the one-particle GFs in the integral

$$\mathcal{A}^{(11)}_{121'2'}(\omega) = \delta_{11'}\delta_{22'} \int G^+_1(\epsilon) G^-_2(\epsilon - \omega) \frac{\mathrm{d}\epsilon}{2\pi \mathrm{i}} = \frac{\delta_{11'}\delta_{22'}}{\omega - \epsilon_{12}}, \tag{81}$$

where $G^{\pm}_{\lambda}(\epsilon) = (\epsilon - \epsilon_\lambda \pm \mathrm{i}\delta)^{-1}$ (here and further we omit the tilde). This propagator determines the usual Tamm-Dancoff equation.

For the case of magic nuclei, to obtain formulae for the terms containing phonon mixing it is necessary to put $\tilde{\Delta} = 0$ in our Appendix. Graphically this corresponds to the three graphs, each of them is the first term on the left in the graphs for the quantity $K^{11}$ of Appendix, eq. (A.2). We should obtain the case without $\mathrm{GSC_{phon}}$ from these graphs. Acting in analogy with eq. (81) and leaving only a "fixed" particle and hole in them we obtain for example for the second graph:

$$I^{2s}_{1231'}(\omega) = \int \frac{\mathrm{d}\epsilon \mathrm{d}\omega_1}{2\pi \mathrm{i}} G^+_1(\epsilon) G^+_3(\epsilon - \omega_1) D_s(\omega_1) G^+_{1'}(\epsilon) G^-_2(\epsilon - \omega). \tag{82}$$

A similar procedure should be done for the rest of the graphs but, according to our above-mentioned plan to include especially the non-pole term of the GF $D_s$ in the

cross phonon graph, we should add the fourth term. The final result is given by

$$\mathcal{A}^{(11)}_{121'2'}(\omega) = \frac{\delta_{11'}\delta_{22'}}{\omega-\epsilon_{12}} + \frac{1}{(\omega-\epsilon_{12})(\omega-\epsilon_{1'2'})} \times \left[\delta_{22'}\sum_{3,s}\frac{g^s_{13}(g^+)^s_{31'}}{\omega-\epsilon_{32}-\omega_s} + \delta_{11'}\sum_{3,s}\frac{g^s_{2'3}(g^+)^s_{32}}{\omega-\epsilon_{13}-\omega_s} - \sum_s g^s_{11'}(g^+)^s_{2'2}\left(\frac{1}{\omega-\epsilon_{1'2}-\omega_s}+\frac{1}{\omega-\epsilon_{12'}-\omega_s}\right)\right]. \quad (83)$$

This formula corresponds to the results obtained in ref. [2] in Nuclear Field Theory.

We substitute (83) into (80) and restrict ourselves here and further to the case of one phonon mode $s$. Then the secular equation is as follows:

$$\frac{1}{\lambda} = f(\omega) \equiv \sum_{12}\frac{|q_{12}|^2}{\omega-\epsilon_{12}} + \sum_{121'2'}\frac{q_{12}q_{1'2'}}{(\omega-\epsilon_{12})(\omega-\epsilon_{1'2'})} \times \left[\delta_{22'}\sum_3\frac{g^s_{13}g^s_{1'3}}{\omega-\epsilon_{32}-\omega_s} + \delta_{11'}\sum_3\frac{g^s_{2'3}g^s_{23}}{\omega-\epsilon_{13}-\omega_s} - \frac{2g^s_{11'}g^s_{22'}}{\omega-\epsilon_{1'2}-\omega_s}\right]. \quad (84)$$

For non-magic nuclei, the secular equation without GSC, which is analogous to eq. (84) for magic nuclei, is derived in analogy with the previous derivation using the formulae of the Appendix. For nuclei with pairing there are, however, some complications: 1) it is necessary to add new GFs $F^{(1)}$ and $F^{(2)}$, 2) as in the QRPA there are additional transitions —diagonal ones and transitions between levels which are both below or both above the Fermi level, 3) instead of one equation for the vertex $V$ there is a system of two equations for $V$ and $V^{\rm h}$.

In order not to complicate the problem algebraically we use the approximation $V = V^{\rm h}$. This corresponds to the absence of spin forces which are not important, in principle, for electric excitations, see in [39] an analysis based on conservation laws. Then we obtain from eq. (69):

$$\frac{1}{\lambda} = \sum_{121'2'} q_{12}(\mathcal{A}^{11}_{121'2'}(\omega_n) + \mathcal{A}^{12}_{121'2'}(\omega_n))q_{1'2'}\,. \quad (85)$$

Substitution of the corresponding formulae of the Appendix gives the secular equation (without any GSC)

$$\frac{1}{\lambda} = f(\omega) = \sum_{121'2'} q_{12}q_{1'2'}\frac{u_1v_2(u_{1'}v_{2'}+v_{1'}u_{2'})}{\omega-E_{12}} \times \left\{\delta_{11'}\delta_{22'} + \frac{1}{\omega-E_{1'2'}}\left[\delta_{22'}\sum_3\frac{g^s_{13}g^s_{1'3}}{\omega-E_{32}-\omega_s}\mathcal{X}_{11'33} + \delta_{11'}\sum_3\frac{g^s_{2'3}g^s_{23}}{\omega-E_{13}-\omega_s}\mathcal{X}_{22'33} - 2g^s_{11'}g^s_{22'}\frac{1}{\omega-E_{1'2}-\omega_s}\mathcal{X}_{121'2'}\right]\right\}, \quad (86)$$

where

$$\mathcal{X}_{121'2'} = u_1u_2u_{1'}u_{2'} + u_1v_2u_{1'}v_{2'} + v_1u_2v_{1'}u_{2'} + v_1v_2v_{1'}v_{2'}\,, \quad (87)$$

and the sums in eqs. (85) and (86) are free, *i.e.* non-restricted.

To solve this equation it is necessary to know the expression for $g^s_{12}$ in the Tamm-Dancoff approximation for nuclei with pairing. It has the form

$$g^s_{12} = q_{12}\left(\sum_{34}\frac{|q_{34}|^2u_3v_4(u_3v_4+u_4v_3)}{(\omega_s-E_{34})^2}\right)^{-\frac{1}{2}}. \quad (88)$$

It can be easily seen from eq. (86) that for the one-level model the complex configurations (they contain $g^2$ terms) do not give a contribution. The terms corresponding to the cross phonon and insertion graphs cancel each other. Thus, within the one-level model, where there are only diagonal transitions, the particle-phonon coupling gives the $\mathrm{GSC_{phon}}$ contribution only, *i.e.* here we have a pure effect caused by $\mathrm{GSC_{phon}}$. After lengthy algebraic manipulations, we obtain the following simple formula for the terms of the propagators $\mathcal{A}^{11}$ and $\mathcal{A}^{12}$:

$$\mathcal{A}^{11} = \mathcal{A}^{12} \equiv \mathcal{A}^{\rm QRPA}_{1111} + \mathcal{A}^{\rm 1p1h\otimes phon}_{1111} = \frac{4Eu^2v^2}{\omega^2-4E^2} - 2u^2v^2g^2\left\{\frac{1}{(\omega-2E)^2(\omega-2E-\omega_s)} + \frac{1}{(\omega+2E)^2(-\omega-2E-\omega_s)}\right\} \quad (89)$$

(we omitted the index 1 of the quantities $u, v, E$). An analysis of the $\mathrm{GSC_{phon}}$ becomes complicated here by the second-order poles and will be performed elsewhere. From qualitative and quantitative points of view, at this stage it is of much more interest to analyse the phonon coupling effects for non-magic nuclei given by eq. (86).

### 7.2 Two-level model calculations

In order to deal with possible unphysical second-order poles $(\omega-E_{12})^{-2}$ of the function $f(\omega)$ defined in eq. (84) which correspond to diagonal $g^2$ terms $\{1=1', 2=2'\}$ and $\{1=2', 2=1'\}$ in eq. (84) or (86), we adopt the following simple procedure. Our approximation to $f(\omega)$ contains the sum of two terms

$$\alpha_{12}(\omega) = \frac{a}{\omega-E_{12}} + \frac{b}{(\omega-E_{12})^2} = \frac{a}{\omega-E_{12}}\left(1+\frac{b}{a(\omega-E_{12})}\right), \quad (90)$$

which correspond to the (Q)TDA and $g^2$ terms correspondingly. If we had gone beyond the $g^2$ approximation, we would have obtained additional terms of higher order. Therefore, we assume that the expression in brackets is the beginning of the infinite geometrical progression

$$1 + \frac{b}{a(\omega-E_{12})} + \left(\frac{b}{a(\omega-E_{12})}\right)^2 + \dots\,, \quad (91)$$

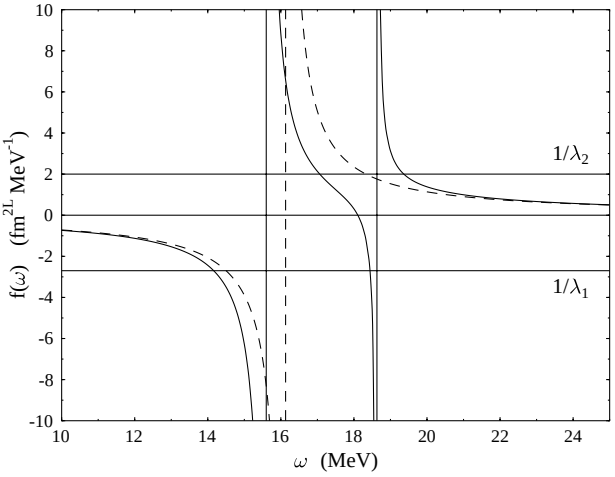



**Fig. 1.** Graphical solution of the secular equation (95). This equation was obtained from eq. (86) in the two-level non-diagonal approximation after the summation procedure (93). The solid curves give the function $f(\omega)$ (94) with phonon coupling, the dashed curves give the corresponding function for the case without phonon coupling. Solutions in both cases are obtained from intersections of the functions $f(\omega)$ with the horizontal straight lines $1/\lambda$ for $\lambda_1 < 0$ and $\lambda_2 > 0$. The vertical straight lines correspond to the phonon coupling poles at $(E_{12}+b/a)$ (at 15.6 MeV) ) and $(E_{12}+\omega_s)$ (at 18.6 MeV), and to the pole at $E_{12}$ (at 16.1 MeV) for the case without phonon coupling. For details see subsect. 7.2.

with the progression factor $q$:

$$q = \frac{b}{a(\omega - E_{12})}\,. \tag{92}$$

Then we perform the summation $\sum_{k=0}^{\infty} q^k$ and obtain the result

$$\alpha_{12}(\omega) \simeq \frac{a}{\omega - E_{12} - b/a}\,, \tag{93}$$

which contains now only the first-order pole, which has been shifted in energy by the amount $b/a$. Of course, there is convergence only if $|q| < 1$. But from the regions of convergence we can continue analytically to yield the result (93). This is illustrated in fig. 1 below.

Let us consider a simple two level model which, in addition, has no diagonal transitions in the limit of absence of complex configurations (two-level non-diagonal approximation). One can easily show that the latter means that the poles of the $(E_{\lambda\lambda} + \omega_s - \omega)^{-1}$ kind are cancelled and, after some simple algebra, we obtain for the function $f(\omega)$ in eq. (86)

$$f(\omega) = |q_{12}|^2(u_1v_2 + u_2v_1)^2 \times \left[\frac{1}{\omega - E_{12}} + \frac{(g_{11} - g_{22})^2}{(\omega - E_{12})^2(\omega - E_{12} - \omega_s)}\right]. \tag{94}$$

Here $g_{11}$ and $g_{22}$ are the amplitudes of a low-lying phonon. We see that for the two-level model considered only diagonal matrix elements of creation amplitudes of the low-lying phonon give a contribution.

After performance of the above-described summation procedure for eq. (94), we obtain the following secular equation:

$$1/\lambda = |q_{12}|^2(u_1v_2 + u_2v_1)^2 \times \left[\frac{a}{\omega - E_{12} - b/a} + \frac{(g_{11} - g_{22})^2}{\omega_s^2(\omega - E_{12} - \omega_s)}\right], \tag{95}$$

where $a = 1 - \omega_s^{-1}(g_{11} - g_{22})^2, b = -\omega_s^{-1}(g_{11} - g_{22})^2$. Thus, as a result, two first order poles at $\omega = E_{12} + b/a$ and $\omega = E_{12} + \omega_s$ are obtained.

In our calculations we put $\tilde{\Delta}_\lambda = \Delta_\lambda$, $\tilde{\epsilon}_\lambda = \epsilon_\lambda$. To demonstrate a realistic pairing effect we have taken $\Delta = 1$ MeV and the low-lying single-particle energies in eq. (88) $\epsilon_\lambda - \mu = -1$ MeV and 2 MeV. Correspondingly, a low-lying phonon with energy $\omega_s = 2.5$ MeV has been assumed. The quantities $q_{\lambda_1\lambda_2}$ were taken of order 1 fm$^L$ where $L$ is the multipolarity of transitions, in particular, in eq. (95) the $q_{12}$ value is 2.1 fm$^L$. This has been done in order to satisfy the conditions that the $g^2$ approximation should be meaningful, but a noticeable value of the energy shift $b/a$ should result. One should note that, according to eq. (88), the order of the $q_{\lambda_1\lambda_2}$ value is not important.

The solution of eq. (95) are obtained graphically from intersections of the function $f(\omega)$ plot with the straight line $1/\lambda$. In order to simplify the analysis and imitate a giant resonance we took a large single-particle difference in eq. (95) for two levels $\epsilon_\lambda - \mu = -10$ MeV and 6 MeV so that $\epsilon_{12} = 16$ MeV. Diagonal matrix elements of the phonon creation amplitude calculated from eq. (88), where the four single-particle levels have been included, are $g_{11} = 0.24$ MeV and $g_{22} = 1.2$ MeV. These four levels at the energies $\epsilon_\lambda - \mu = -1$, 2, $-10$, and 6 MeV have been used to correspond realistically to our low-lying phonon at 2.5 MeV, on the one hand, and to have matrix elements $g_{\lambda_1\lambda_2}$ for the two high-lying levels under consideration, on the other hand.

The graphical solution of eq. (95) is given in fig. 1. The lines $1/\lambda$ were drawn rather arbitrarily to show solutions for both $\lambda < 0$ ("isoscalar resonance") and $\lambda > 0$ ("isovector resonance") We obtained $b/a = -0.5$ MeV. This shift is considerably smaller (in absolute value) than the shift obtained from the mixing of particle-hole configurations ("simple configurations") in TDA (or RPA). In addition to this shift there occurs a redistribution of transition strength. The increase in the number of solutions, as compared with the QRPA case, and the redistribution of strength correspond to the spreading width $\Gamma^{\downarrow}$ of giant resonances in non-magic nuclei which, as it is known, is one of the main contributions to the total width for heavy nuclei. A more detailed and realistic model analysis will be performed elsewere.

## 8 Conclusion

In this work a microscopic approach has been formulated for non-magic nuclei which is a generalization of the QRPA, to be exact of the standard TFFS. It takes into account:

1. complex configurations of the two-quasiparticle–phonon type which makes it possible to go beyond the QRPA due to explicit treatment of the quasiparticle-phonon interaction;
2. the single-particle continuum which makes it possible to obtain a realistic envelope of the resonance without using a smearing parameter and, in addition, to decrease strongly numerical difficulties which is especially important for the problem of accounting for complex configurations;
3. ground-state correlations caused not only by the QRPA configurations but by the more complex ones which are more important and interesting physically than the QRPA ground-state correlations;
4. non-separable particle-hole and particle-particle interactions, the parameters of which are mainly known;
5. the quasiparticle-phonon interaction in the particle-particle channel, including a refinement of the phenomenological pairing gap from this interaction. The latter means taking explicitely into account the main part of the quasiparticle-phonon mechanism of nuclear pairing (in addition to the usual mechanism of BCS type).

Thus, when applied to the theory of giant multipole resonances in non-magic nuclei, the approach explicitely contains the three known mechanisms of resonance damping: decay via particle-hole configurations of the discrete spectrum (analog of Landau damping), decay via particle-hole configurations with a particle in the continuum (escape width) and decay via more complicated confiurations of two-quasiparticle–phonon type (spreading width).

Two features of the theory —account for the single-particle continuum and making use of the known interaction parameters— allow to hope for successful application of the theory to calculate unstable non-magic nuclei including those with nucleon separation energy near zero. It would be very important to check the universality of the Landau-Migdal interaction in exotic nuclei to understand where this universality is valid.

The specificity of non-magic nuclei in our approach is not only the consistent inclusion of anomalous mass operators $M^{(1)}$, $M^{(2)}$ (eq. (2)) to describe the nuclear excitations but also an improvement of nuclear pairing theory by taking explicitely into account the refinement of the phenomenological gap. The quantitative role of these effects in the particle-particle channel have been already shown in calculations of energies and spectroscopic factors for low-lying excitation in $^{119}$Sn and $^{121}$Sn [30]. It is very interesting to find other consequences of these effects, in particular, to understand better the role and mechanisms of nuclear pairing.

Thus in the approach developed, two ingredients — ground-state correlations and the refinement of the phenomenological gap connected with an improved pairing theory— should be important for the description of low-lying excitations in non-magic nuclei. The possibilities of the modern gamma spectrometers [18] allow to study these effects.

In conclusion, one can hope that the microscopic approach developed, which together with the theory for magic nuclei [5, 10–16] can be called the Extended Theory of Finite Fermi Systems, corresponds to the modern status of low-energy experimental physics including both the giant resonances and the results obtained using modern $\gamma$-detector arrays. On the one hand, it is an improved theory with some new ingredients and simultaneous treatment of several other ones known before. On the other hand, the numerical realization of the approach should not be too involved to apply it to many stable and unstable non-magic nuclei. We cannot yet present numerical results referring to actual nuclei. This will be done in future work.

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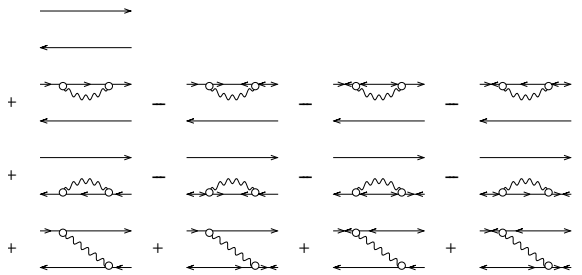
## Appendix A. Propagators of the system for the effective fields in non-magic nuclei taking into account QRPA and two-quasiparticle–phonon configurations with $\mathbf{GSC_{phon}}$

We use the one-quasiparticle Green functions determined by (28), the one-phonon Green function

$$D_s(\omega) = \frac{1}{\omega - \omega_s + \mathrm{i}\delta} - \frac{1}{\omega + \omega_s - \mathrm{i}\delta} \qquad (A.1)$$

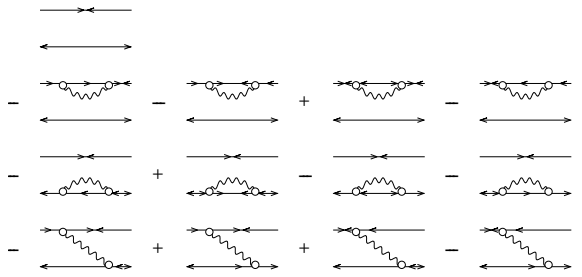
and omit the tilde "~" in all the Appendices. The graphic representation for the quantities $K^{ik}$ is the following:

$K^{11} =$



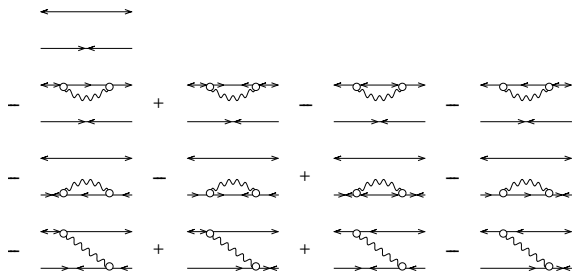
(A.2)

$K^{12} =$



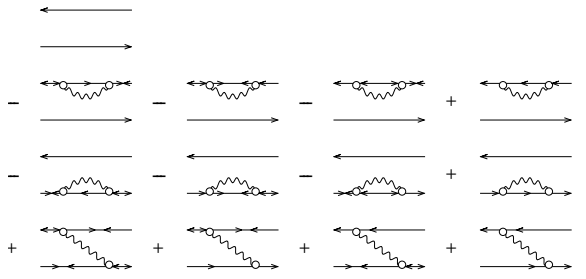
(A.3)

$K^{21} =$



(A.4)

$K^{22} =$



(A.5)

Let us introduce an additional notation for the terms of propagator corresponding to various graphs in this representation. In the quantities $\mathcal{A}^{ik;l}$ the additional index $l = 0$–12 denotes the number of the graph: $l = 0$ corresponds to the QRPA one, $l = 1$–4 is for the "left" insertion graphs, $l = 5$–8 for the "right" insertion graphs and $l = 9$–12 for the phonon exchange graphs, *e.g.*

$$\mathcal{A}^{ik} = \sum_{l=0\text{-}12} \mathcal{A}^{ik;l}\,. \tag{A.6}$$

The QRPA components of propagator are given by the known expressions:

$$\begin{aligned}
\mathcal{A}^{11;0}_{121'2'}(\omega) &= \delta_{11'}\delta_{22'}\int\frac{\mathrm{d}\epsilon}{2\pi\mathrm{i}}G_1(\epsilon)G_2(\epsilon-\omega) = \\
&\quad -\delta_{11'}\delta_{22'}\Big(\frac{u_1^2v_2^2}{E_{12}-\omega}+\frac{v_1^2u_2^2}{E_{12}+\omega}\Big); \\
\mathcal{A}^{12;0}_{121'2'}(\omega) &= \delta_{11'}\delta_{22'}\int\frac{\mathrm{d}\epsilon}{2\pi\mathrm{i}}F_1^{(2)}(\epsilon)F_2^{(1)}(\epsilon-\omega) = \\
&\quad \delta_{11'}\delta_{22'}\frac{2E_{12}u_1v_1u_2v_2}{E_{12}^2-\omega^2},
\end{aligned} \tag{A.7}$$

where $E_{12} = E_1 + E_2$. The first term with "left" insertion graph corresponding to the second graph in $K^{11}$ has the form

$$\mathcal{A}^{11;1}_{121'2'}(\omega) = -\delta_{22'}\sum_{3,s} g^s_{13}(g^{s+})_{31'}I^{(11;1)s}_{121'3}(\omega); \tag{A.8}$$

$$\begin{aligned}
&I^{(11;1)s}_{121'3}(\omega) = \\
&\int\frac{\mathrm{d}\epsilon\mathrm{d}\omega_1}{(2\pi\mathrm{i})^2}G_1(\epsilon)G_3(\epsilon-\omega_1)G_{1'}(\epsilon)G_2(\epsilon-\omega)D_s(\omega_1) = \\
&v_3^2G_2^{\mathrm{h}}(E_3+\omega+\omega_s)\left[\frac{u_1^2G_{1'}(E_1)}{E_{13}+\omega_s}+\frac{u_{1'}^2G_1(E_{1'})}{E_{1'3}+\omega_s}\right] \\
&+u_3^2G_2(E_3-\omega+\omega_s)\left[\frac{v_1^2G^{\mathrm{h}}_{1'}(E_1)}{E_{13}+\omega_s}+\frac{v_{1'}^2G^{\mathrm{h}}_1(E_{1'})}{E_{1'3}+\omega_s}\right] \\
&+v_2^2G^{\mathrm{h}}_{3s}(E_2-\omega)\left[\frac{u_1^2G_{1'}(E_1)}{E_{12}-\omega}+\frac{u_{1'}^2G_1(E_{1'})}{E_{1'2}-\omega}\right] \\
&+u_2^2G_{3s}(E_2+\omega)\left[\frac{v_1^2G^{\mathrm{h}}_{1'}(E_1)}{E_{12}+\omega}+\frac{v_{1'}^2G^{\mathrm{h}}_1(E_{1'})}{E_{1'2}+\omega}\right],
\end{aligned} \tag{A.9}$$

where

$$G_{3s}(\epsilon) = \frac{u_3^2}{\epsilon-E_{3s}}+\frac{v_3^2}{\epsilon+E_{3s}}\,,\quad E_{3s} = E_3+\omega_s\,. \tag{A.10}$$

The corresponding term with the "right" insertion graph in $K^{11}$ is expressed through the previous one

$$\mathcal{A}^{11;5}_{121'2'}(\omega) = -\delta_{11'}\sum_{3,s} g^s_{2'3}(g^{s+})_{32}I^{(11;5)s}_{122'3}(\omega); \tag{A.11}$$

$$I^{(11;5)s}_{122'3}(\omega) = I^{(11;1)s}_{212'3}(-\omega)\,. \tag{A.12}$$

The term with phonon exchange has the form

$$\mathcal{A}^{11;9}_{121'2'}(\omega) = -\sum_{s} g^s_{11'}(g^{s+})_{2'2}I^{(11;9)s}_{121'2'}(\omega); \tag{A.13}$$

$$\begin{aligned}
&I^{(11;9)s}_{121'2'}(\omega) = \int\frac{\mathrm{d}\epsilon\mathrm{d}\omega_1}{(2\pi\mathrm{i})^2} \\
&\times G_1(\epsilon)G_2(\epsilon-\omega)G_{1'}(\epsilon-\omega_1)G_{2'}(\epsilon-\omega_1-\omega)D_s(\omega_1) = \\
&v_2^2G^{\mathrm{h}}_1(E_2-\omega)\left[\frac{u_{2'}^2G_{1'}(E_{2'}+\omega)}{E_{22'}+\omega_s}+\frac{u_{1'}^2G_{2'}(E_{1'}-\omega)}{E_{21'}-\omega+\omega_s}\right] \\
&+u_2^2G_1(E_2+\omega)\left[\frac{v_{2'}^2G^{\mathrm{h}}_{1'}(E_{2'}-\omega)}{E_{22'}+\omega_s}+\frac{v_{1'}^2G^{\mathrm{h}}_{2'}(E_{1'}+\omega)}{E_{21'}+\omega+\omega_s}\right] \\
&+v_1^2G^{\mathrm{h}}_2(E_1+\omega)\left[\frac{u_{2'}^2G_{1'}(E_{2'}+\omega)}{E_{12'}+\omega+\omega_s}+\frac{u_{1'}^2G_{2'}(E_{1'}-\omega)}{E_{11'}+\omega_s}\right] \\
&+u_1^2G_2(E_1-\omega)\left[\frac{v_{2'}^2G^{\mathrm{h}}_{1'}(E_{2'}-\omega)}{E_{12'}-\omega+\omega_s}+\frac{v_{1'}^2G^{\mathrm{h}}_{2'}(E_{1'}+\omega)}{E_{11'}+\omega_s}\right]
\end{aligned} \tag{A.14}$$

As the expressions for the normal and the anomalous Green functions contain the same poles and differ only by the occupation numbers (residues) and signs,

$$\begin{aligned}
G_\lambda(\epsilon) &= \frac{u_\lambda^2}{\epsilon-E_\lambda+\mathrm{i}\delta}+\frac{v_\lambda^2}{\epsilon+E_\lambda-\mathrm{i}\delta}; \\
F_\lambda(\epsilon) &= -u_\lambda v_\lambda\left[\frac{1}{\epsilon-E_\lambda+\mathrm{i}\delta}-\frac{1}{\epsilon+E_\lambda-\mathrm{i}\delta}\right],
\end{aligned} \tag{A.15}$$

one may obtain $\mathcal{A}^{11;2}$, $\mathcal{A}^{11;3}$ from $\mathcal{A}^{11;1}$ by formal substitutions. To get the expression for $\mathcal{A}^{11;2}$, we should perform the following substitutions:

$$\left.\begin{array}{l} u^2_{1'} \to -u_{1'}v_{1'} \\ v^2_{1'} \to u_{1'}v_{1'} \\ u^2_3 \to -u_3v_3 \\ v^2_3 \to u_3v_3 \end{array}\right\} . \quad (A.16)$$

Thus, the quantity $\mathcal{A}^{11;2}$ is given by

$$\mathcal{A}^{11;2}_{121'2'}(\omega) = -\delta_{22'} \sum_{3,s} g^s_{13}(g^{s+})_{31'} I^{(11;2)s}_{121'3}(\omega)\,; \quad (A.17)$$

$$\begin{aligned} & I^{(11;2)s}_{121'3}(\omega) = \\ & u_3v_3G^{\rm h}_2(E_3+\omega+\omega_s)\left[\frac{u^2_1F_{1'}(E_1)}{E_{13}+\omega_s} - \frac{u_{1'}v_{1'}G_1(E_{1'})}{E_{1'3}+\omega_s}\right] \\ & - u_3v_3G_2(E_3-\omega+\omega_s)\left[\frac{v^2_1F_{1'}(E_1)}{E_{13}+\omega_s} + \frac{u_{1'}v_{1'}G^{\rm h}_1(E_{1'})}{E_{1'3}+\omega_s}\right] \\ & + v^2_2F_{3s}(E_2-\omega)\left[\frac{u^2_1F_{1'}(E_1)}{E_{12}-\omega} - \frac{u_{1'}v_{1'}G_1(E_{1'})}{E_{1'2}-\omega}\right] \\ & + u^2_2F_{3s}(E_2+\omega)\left[\frac{v^2_1F_{1'}(E_1)}{E_{12}+\omega} + \frac{u_{1'}v_{1'}G^{\rm h}_1(E_{1'})}{E_{1'2}+\omega}\right] . \quad (A.18) \end{aligned}$$

To obtain the expression for $\mathcal{A}^{11;3}$ we should substitute

$$\left.\begin{array}{l} u^2_1 \to -u_1v_1 \\ v^2_1 \to u_1v_1 \\ u^2_3 \to -u_3v_3 \\ v^2_3 \to u_3v_3 \end{array}\right\} . \quad (A.19)$$

The term $\mathcal{A}^{11;4}$ contains the function $G^{\rm h}$

$$G^{\rm h}_\lambda(\epsilon) = G_\lambda(-\epsilon) = \frac{-v^2_\lambda}{\epsilon - E_\lambda + {\rm i}\delta} + \frac{-u^2_\lambda}{\epsilon + E_\lambda - {\rm i}\delta}\,, \quad (A.20)$$

so it can be obtained by the substitution

$$\left.\begin{array}{l} u^2_1 \to -u_1v_1 \\ v^2_1 \to u_1v_1 \\ u^2_{1'} \to -u_{1'}v_{1'} \\ v^2_{1'} \to u_{1'}v_{1'} \\ u^2_3 \to -v^2_3 \\ v^2_3 \to -u^2_3 \end{array}\right\} . \quad (A.21)$$

Other components of the propagator can be obtained by the same way from the two "base" expressions for $\mathcal{A}^{11;1}_{121'2'}(\omega)$ and $\mathcal{A}^{11;9}_{121'2'}(\omega)$.

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